第一届“粤港澳”核物理论坛

Twisted/vortex particles in magnetic fields

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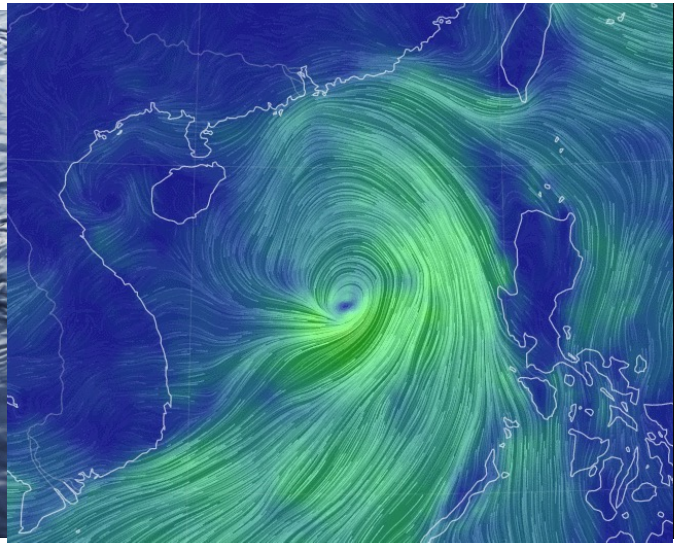
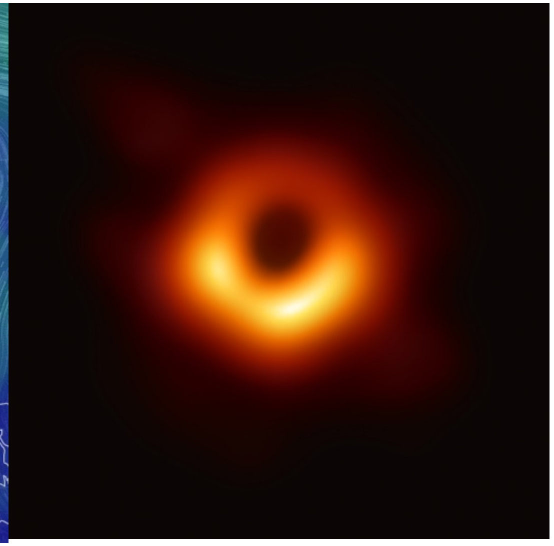
- Vortex particles(electron)—an introduction

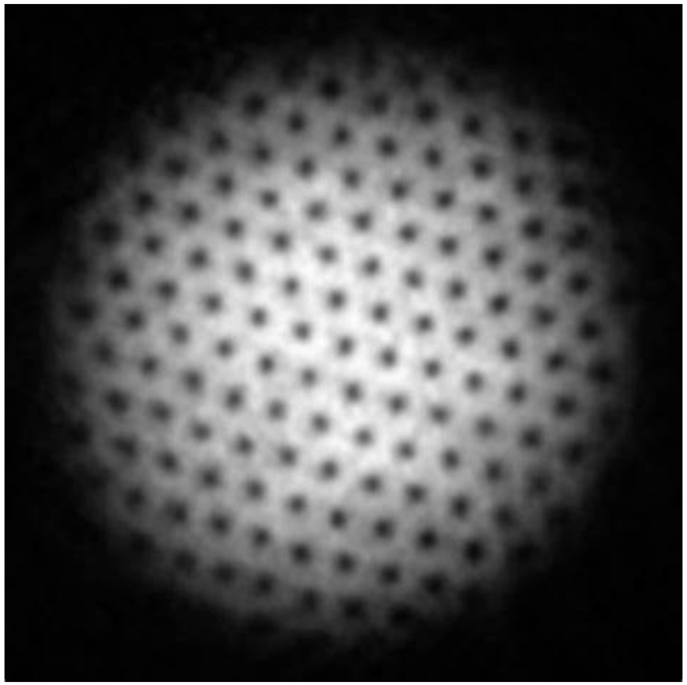
- Theoretical descriptions
  - Schrödinger equation, Dirac equation in FW presentation
  - Bessel beam, Lagueree-Gauss beam

- Dynamics in electromagnetic fields
  - Landau beam state
  - Vortex particle production in magnetic fields

- Summary

$$v(r) \sim \frac{1}{r}$$



Quantum vortex in superfluid/ superconductor

$$\psi(r) = \rho(r)e^{in\varphi}$$

$$v_s = \frac{n\hbar}{m}\nabla\varphi$$

circulation quantization:

$$\oint v_s \cdot dr = \frac{n\hbar}{m}2\pi$$

A quantum state of (free) electron

Plane wave?

Solution of the Schrodinger/Dirac equation

Vortex/twisted electron

Bessel beam, Laguerre-Gauss beam

- Non–plane-wave solutions of wave equation with helicoidal wave fronts
- Intrinsic OAM with respect to the average propagation direction
  — a new degree of freedom
- Effect of a single particle

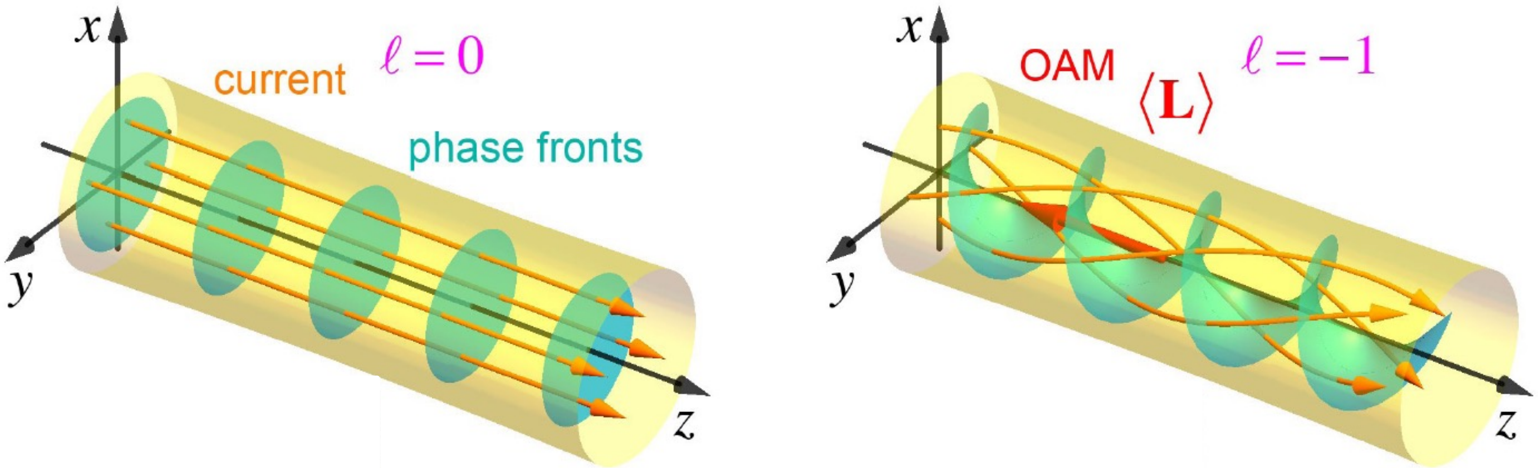
Vortex State

Cylindrical waves, singular phase vortices with winding number :

$$\Psi_l = r^l e^{il\varphi}$$

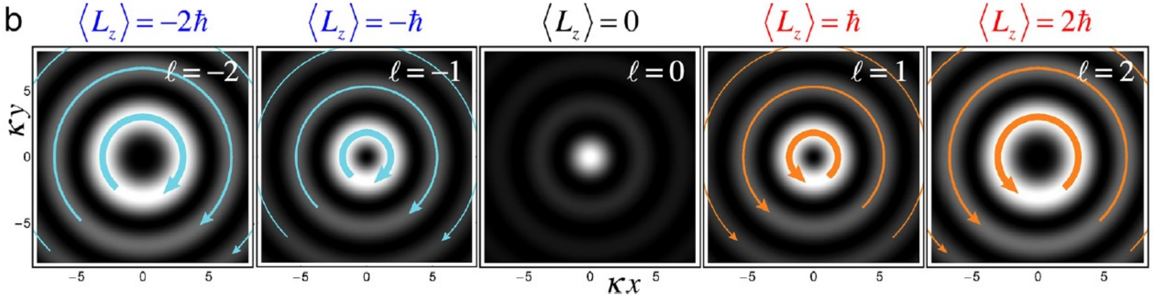
$$\boxed{L_z = \hbar\ell}$$

well-defined OAM



helical phase fronts, and spiraling currents

$|\Psi|^2$



Bliokh, et al, Phys. Rept. 690 (2017) 1

Bessel beam and Laguerre-Gauss beam

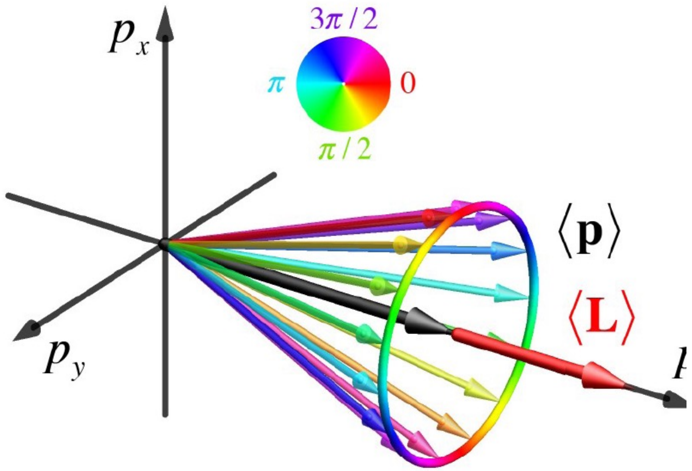
Monochromatic solutions of wave equation:

plane waves: definite $w, \vec{k}$

Bessel beams: definite $w,\ k_z,\ k_\perp,\ l$

Laguerre-Gauss beam : definite $w,\ k_z,\ l,\ a$

in cylindrical coordinates



Bessel beam

$$\Psi_l(r,z) \sim e^{-iwt} e^{ik_z z} e^{il\varphi} J_l(k_\perp r)$$

Laguerre-Gauss beam

$$\Psi_l(r,z) \sim e^{-iwt} e^{ik_z z} e^{il\varphi} \left(\frac{\sqrt{r}}{a(z)}\right)^l e^{-\frac{r^2}{a(z)^2}} L_n^l \left(-\frac{r^2}{a(z)^2}\right) e^{i\phi}$$

Experimental progresses

- Vortex states of optical photons were produced back in 1990's

  Vortex states of X rays in 2013

  Applications: trapped nanoparticles, quantum information, interaction with matter, microscopy etc.

- Vortex electron was produced in 2010 on electron microscopes

  Well demonstrated at low energies

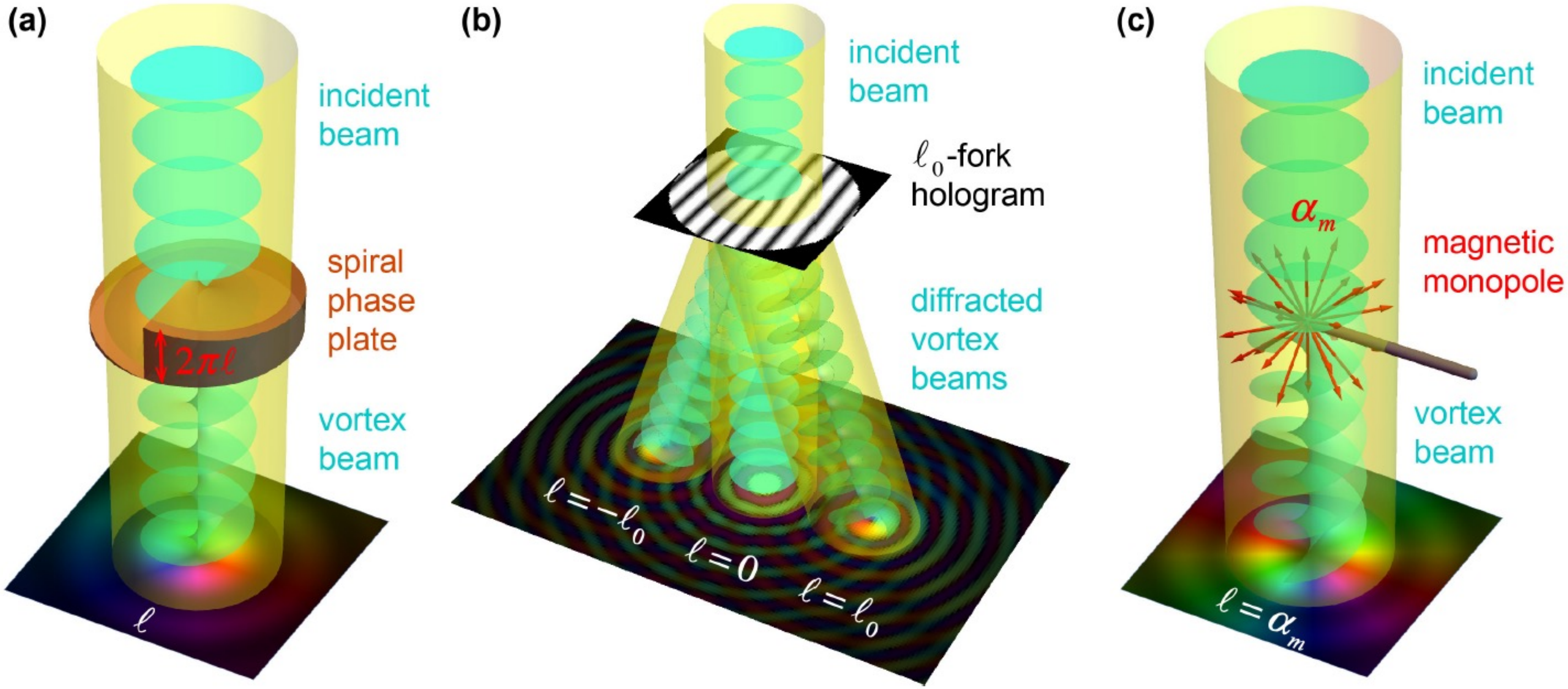
  - how to put high-energy electrons in vortex states
  - how to accelerate low-energy vortex electrons to higher energies

  None has been tried in experiment.

  Applications: atomic-scale probe of magnetic properties and other material studies; Control of isomeric nuclear decay by using electrons in nuclear physics; Doing spin physics with twisted electron…

- Neutrons in 2015, Atom and molecules last year

transmission electron microscope (TEM)



magnetic needle

E ~ 300 KeV, OAM up to 1000

Uchida, Tonomura, Nature 464, 737 (2010);
Verbeeck, Tian, Schattschneider, Nature 467, 301 (2010);
McMorran et al, Science 331, 192 (2011)

Many successes, but far from enough

How to obtain high energy vortex states

1. First accelerate, then twist: a high energy particle in an approximately plane wave state with sufficient transverse coherence length passes through a device which imparts a phase vortex.
Possible, but the usual twisting devices become impractical.

2. First twist, then accelerate: a low-energy vortex state is injected in a linear accelerator or a storage ring where its energy is increased without destroying the phase vortex.
Optimistic, but requires dedicated numerical studies and proof-of-principle experiments.

3. Transfer the OAM and/or energy from an initial particle to the final one through a high-energy collision process itself.

I. Igor, arXiv:2205.00412

Dynamics of vortex particle in fields

a relativistic description for twist electron in vacuum/electromagnetic filed in Foldy-Wouthuysen representation of Dirac equation.

Dirac Hamiltonian

$$(\beta m + \alpha \cdot \mathbf{p})\Psi = E\Psi$$

$$H' = e^{iS} H_{Dirac} e^{-iS} = \beta\sqrt{m^2 + p^2} = \beta E$$

Dirac equation

- ✓ (block) diagonal Hamiltonian
- ✓ equation takes the Schrodinger form
  defined operators have their counterparts in classical physics
- ✓ energy, momentum and velocity has clear and definite relations
- ✓ does not mix the positive energy solutions with negative energy
  In Dirac representation, the operators(spin, position…) definitions are controversial

Zitterbewegung

One of the most important feature of the twisted electron is **Intrinsic OAM**

We proposed a electron-cloud/charged centroid model to describe a twisted electron, and define the intrinsic OAM for the first time

- ◆intrinsic OAM ---from the rotation of the charged cloud.
- ◆extrinsic OAM ---by the motion of the center of charge of the electron.

In FW representation we have

$$\mathcal{H}_{\mathrm{FW}} = \beta\epsilon + e\Phi - \beta\frac{e}{4}\left[\frac{1}{\epsilon}\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}\frac{1}{\epsilon}\right] + \frac{e}{4}\left\{\frac{1}{\epsilon^2}\boldsymbol{L}\cdot[\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R})] - [\boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}']\cdot\boldsymbol{L}\frac{1}{\epsilon^2}\right\}.$$

PRL 122 (2019)063201

Relativistic quantum dynamics in electromagnetic field

$$\boldsymbol{F} = \frac{d\boldsymbol{\pi}'}{dt} = \frac{\partial \boldsymbol{\pi}'}{\partial t} + i[\mathcal{H}_{\mathrm{FW}}, \boldsymbol{\pi}']$$
$$= e\boldsymbol{E}(\boldsymbol{R}) + \beta\frac{e}{4}\left\{\frac{1}{\epsilon}, (\boldsymbol{\pi}' \times \boldsymbol{B}(\boldsymbol{R}) - \boldsymbol{B}(\boldsymbol{R}) \times \boldsymbol{\pi}')\right\} + \boldsymbol{F}_{SGl}.$$

$\mathbf{F}_{\mathbf{SGl}}$ is the Stern-Gerlach-like force by the non-uniform field

$$\boldsymbol{F}_{\mathrm{SGl}} = \beta\frac{e}{4}\left\{\frac{1}{\epsilon}\nabla[\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R})] + \nabla[\boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}]\frac{1}{\epsilon}\right\}$$
$$-\frac{e}{4}\left\{\frac{1}{\epsilon^2}\nabla(\boldsymbol{L}\cdot[\boldsymbol{\pi}' \times \boldsymbol{E}(\boldsymbol{R})]) - \nabla([\boldsymbol{E}(\boldsymbol{R}) \times \boldsymbol{\pi}']\cdot\boldsymbol{L})\frac{1}{\epsilon^2}\right\}.$$

Evolutions of intrinsic OAM

$$\frac{d\boldsymbol{L}}{dt} = i[\mathcal{H}_{\mathrm{FW}}, \boldsymbol{L}] = \frac{1}{2}(\boldsymbol{\Omega} \times \boldsymbol{L} - \boldsymbol{L} \times \boldsymbol{\Omega}),$$
$$\boldsymbol{\Omega} = -\beta\frac{e}{4}\left\{\frac{1}{\epsilon}, \boldsymbol{B}(\boldsymbol{R})\right\}$$
$$+\frac{e}{4}\left[\frac{1}{\epsilon^2}\boldsymbol{\pi}' \times \boldsymbol{E}(\boldsymbol{R}) - \boldsymbol{E}(\boldsymbol{R}) \times \boldsymbol{\pi}'\frac{1}{\epsilon^2}\right].$$

Intrinsic OAM looks very like spin

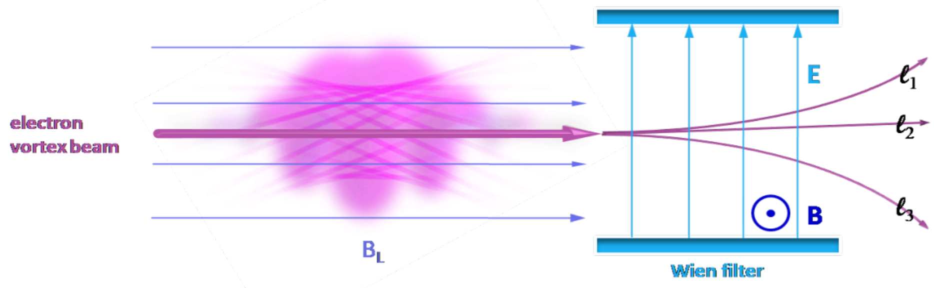
But unlike spin, has no Thomas precession term

Manipulating twisted electron beams

PRL 121 (2018)043202, 119 (2017) 243903

1. Separations of beams with different OAM

extract a beam with a
needed orbital polarization



2. Freezing the intrinsic OAM in electromagnetic fields

$$\frac{d\boldsymbol{N}}{dt} = \boldsymbol{\omega} \times \boldsymbol{N}, \qquad \boldsymbol{\omega} = -\frac{e}{mc\gamma}\left(\boldsymbol{B} - \frac{\boldsymbol{N} \times \boldsymbol{E}}{\beta}\right) \qquad \boldsymbol{B} = \left(\frac{2}{\beta^2} - 1\right)\boldsymbol{\beta} \times \boldsymbol{E}$$

3. Rotate OAM relative to the momentum direction

change direction of OAM but keep the direction of momentum
desirable and even necessary when the beam is confined in a storage ring or trap

$$\frac{d\mathbf{L}}{dt} = -\frac{e}{2mc\gamma}\mathbf{B} \times \mathbf{L}$$

Quantum states of vortex electron in magnetic fields

1. Landau states

K.Y. Bliokh, et al, Phys. Rev. X 2 (2012) 041011.

$$\psi^L_{\ell,n} \propto \left(\frac{r}{w_m}\right)^{|\ell|} L_n^{|\ell|}\left(\frac{2r^2}{w_m^2}\right) \exp\left(-\frac{r^2}{w_m^2}\right) \exp\left[i(\ell\varphi + k_z z)\right]$$

Not really a **twisting state**, it's a state of **Landau levels with OAM**

$$w_m = \frac{2}{\sqrt{|e|B}},$$

transverse magnetic length

- No structure along the magnetic field direction
- Not consistent with free twisted electron beams, in the weak-field limit

$$\Psi = \mathbb{A}\,\exp(i\Phi), \qquad \int \Psi^\dagger \Psi r\,dr\,d\phi = 1,$$

$$\mathbb{A} = \frac{C_{n\ell}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|\ell|} L_n^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right)\eta,$$

$$w(z) = w_0\sqrt{1+\frac{z^2}{z_R^2}}, \quad R(z) = z + \frac{z_R^2}{z}, \quad z_R = \frac{kw_0^2}{2},$$

$$\Phi = \ell\phi + \frac{kr^2}{2R(z)} - \Phi_G(z),$$

$$\Phi_G(z) = N\arctan\left(\frac{z}{z_R}\right), \quad N = 2n + |\ell| + 1,$$

Quantum states of vortex electron in magnetic fields

PRA 103, L010201 (2021)
JPG 47 (2020) 5, 055003

2. Twisting Landau states

A **new** solution of Dirac equation in paraxial condition

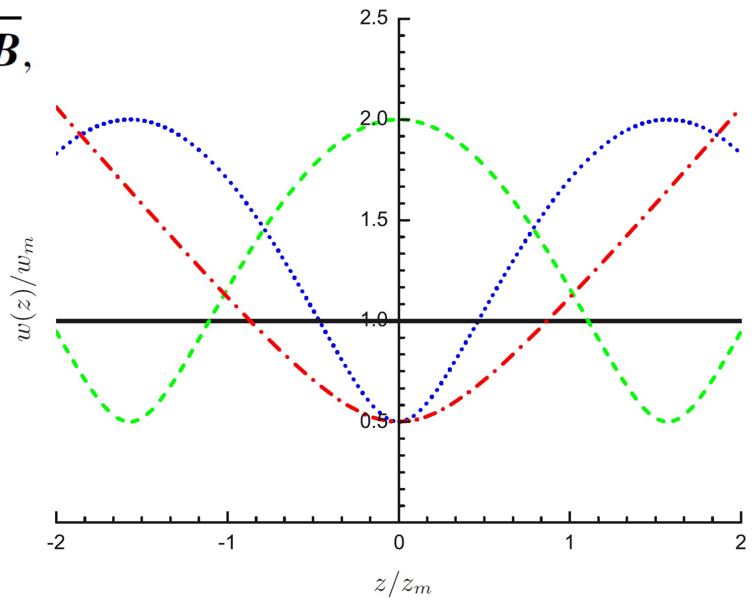
$$i\frac{\partial \Psi_{\mathrm{FW}}}{\partial t} = \mathcal{H}_{\mathrm{FW}}\Psi_{\mathrm{FW}}, \quad \mathcal{H}_{\mathrm{FW}} = \beta\sqrt{m^2 + \boldsymbol{\pi}^2 - e\boldsymbol{\Sigma}\cdot\boldsymbol{B}},$$

$$w(z) = w_0\sqrt{\frac{1}{2}\left[1 + \frac{w_m^4}{w_0^4} - \left(\frac{w_m^4}{w_0^4} - 1\right)\cos\frac{2z}{z_m}\right]}$$

$$= w_0\sqrt{\cos^2\frac{z}{z_m} + \frac{w_m^4}{w_0^4}\sin^2\frac{z}{z_m}}, \quad z_m = \frac{kw_m^2}{2},$$

$$R(z) = kw_m^2\frac{\cos^2\frac{z}{z_m} + \frac{w_m^4}{w_0^4}\sin^2\frac{z}{z_m}}{\left(\frac{w_m^4}{w_0^4} - 1\right)\sin\frac{2z}{z_m}},$$

$$\Phi_G(z) = N\arctan\left(\frac{w_m^2}{w_0^2}\tan\frac{z}{z_m}\right) + \frac{(\ell + 2s_z)z}{z_m}.$$



- with structures along the magnetic field direction
- consistent with free twisted electron beams in the weak-field limit

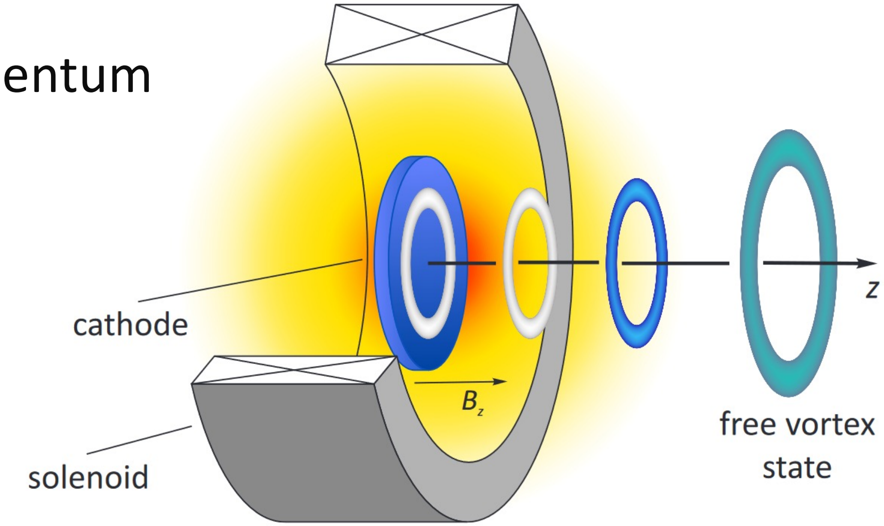
Production of vortex electrons in magnetic fields

A new idea (seems promising) to generate vortex electrons

Quantum Busch theorem:
conservation of the canonical angular momentum in axially symmetric field

$$\tilde{L} = x\tilde{p}_y - y\tilde{p}_x = rp_\theta + qrA_\theta$$

$$\approx \underbrace{xp_y - yp_x}_{\text{Kinetic OAM}} + \underbrace{\frac{qB_z}{2}(x^2+y^2)}_{\text{flux}}$$



A cathode placed inside a solenoid can emit electrons along the solenoid axis

Electrons emitted with L = 0 on the cathode carry the canonical angular momentum,

$$\tilde{L} = \frac{q\Phi}{2\pi}$$

which turns into the kinetic angular momentum in the field free region.

$$\tilde{L} = L = xp_y - yp_x$$

$$10^8\hbar$$

OAM-dominated electron beams @ FNAL

Y.-E Sun, et al, Phys. Rev. ST Accel. Beams 7, 123501(2004).

Charged particles produced in a magnetic field and penetrating from it are twisted

$$w(z) = w_0\sqrt{\frac{1}{2}\left[1+\frac{w_m^4}{w_0^4}-\left(\frac{w_m^4}{w_0^4}-1\right)\cos\frac{2z}{z_m}\right]}$$

- relativistic quantum mechanics requires continuous wave function on the boundary
- the total canonical OAM conserves

Radial magnetic field in real solenoid

$$B_R(R,\phi,z) = -\frac{R}{2}\frac{\partial B_z}{\partial z} \equiv -\frac{R}{2}B_z', \qquad B_\phi(R,\phi,z)=0,$$

$$B_z(R,\phi,z) = B_z(0,\phi,z) - \frac{R^2}{4}\frac{\partial^2 B_z}{\partial z^2} \equiv B_z(0,\phi,z) - \frac{R^2}{4}B_z''.$$

leads to a azimuthal kinetic momentum change

$$\frac{d\boldsymbol{\pi}}{dt} = \beta\frac{e}{4}\left\{\frac{1}{\epsilon'},(\boldsymbol{\pi}\times\boldsymbol{B}-\boldsymbol{B}\times\boldsymbol{\pi})\right\}$$

$$\epsilon' = \sqrt{m^2+\boldsymbol{\pi}^2-e\boldsymbol{\Sigma}\cdot\boldsymbol{B}},$$

$\longrightarrow \Delta\pi_\phi$

Canonical OAM does not conserve

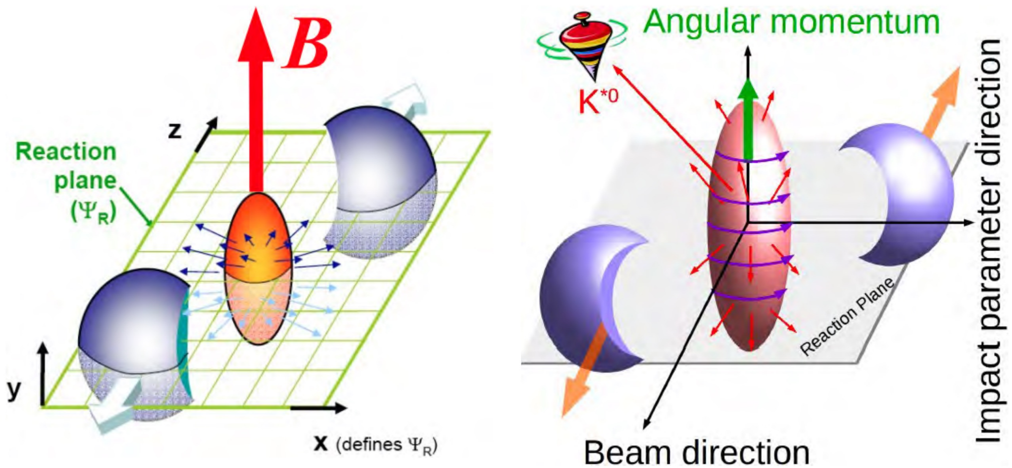
Magnetic fields and vorticity in Heavy Ion Collisions

Strongest magnetic field

10^{14}-10^{16} T

Most vortical fluid

$J \sim 10^6 \hbar$



There are possibilities to find twisted particles HIC

Charged particles emitted within

magnetic field： ~ 10^{15} T ~ 10 m_π^2

Pb-Pb collision @LHC

area of magnetic field： ~ 10 fm

Quantum Busch theorem：

Twisted particle can be generated

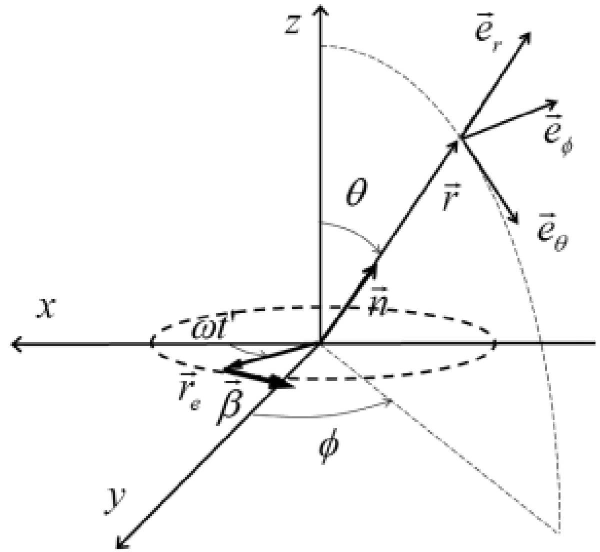
$$L_z^{max} = \frac{|e|Br_{max}^2}{2} \longrightarrow |\ell|_{max} \sim 10^2$$

arXiv:2112.12404

Twisted radiation from a charge in spiral motion

M. Katoh, et al, PRL 118, 094801 (2017)

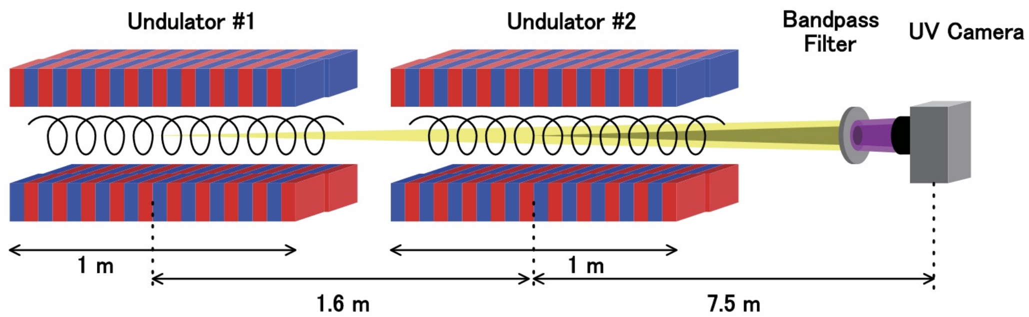
Free electrons in circular motion emit twisted photons carrying well-defined OAM along the axis of the circulation

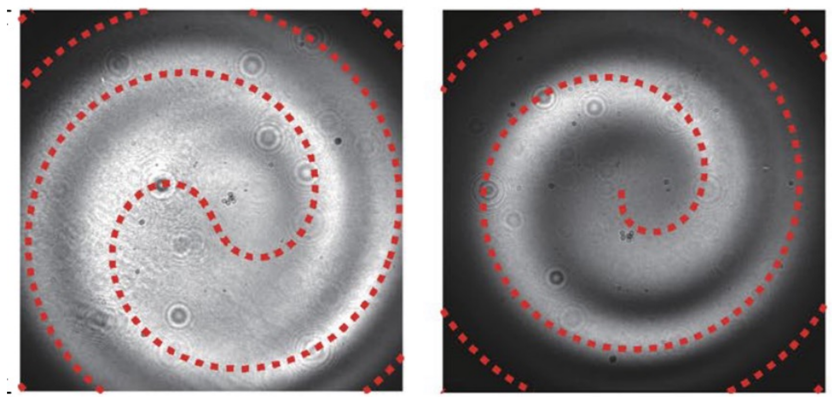


each photon with energy $\hbar l\omega$ carries $l\hbar$ AM

Classical radiation spectrum for relativistic electron in circular motion

$$\Omega \sim \gamma^3 \omega$$





@UVSOR-III Synchrotron facility

M. Katoh et al., Sci. Rep. 7, 6130(2017)

Quantun radiation spectrum

$$\hbar\Omega \sim (\varepsilon - \hbar\Omega)\chi, \qquad \chi \approx \frac{\hbar\omega\varepsilon^2}{m^3}$$

$\hbar\Omega \ll \varepsilon$: $$L_z \approx \hbar\gamma^3$$

$\hbar\Omega \sim \varepsilon$: $$L_z = \frac{\hbar\Omega}{\omega} \approx \frac{\gamma m}{\omega}$$

Relativistic particles

$$\gamma \gg 1$$

Twisted X/gamma rays generated from electrons in Landau states under astrophysical magnetic fields ~ 10^{12} G

T. Maruyama, et al., PLB 826 (2022) 136779

- Photons emitted due to electromagnetic interactions at noncentral HIC could be significantly twisted.
- Twisted particles in heavy ion collision?

The resonances produced can be polarized,
even if the colliding vortex particles are unpolarized.

100% polarized vector mesons in unpolarized twisted e+e− annihilation

I. Ivanov, et al., PRL 124, 192001

Summary

- Vortex particles have realized in different experiments and found applications.

- Understanding the dynamics of vortex electrons in electromagnetic fields is a key step for future application of twisted electron in high energy nuclear/particle physics.
  - How to generate twisted electron
  - How to accelerate twisted electron
  - Beam optics systems in linear accelerators or in storage rings are not well known, as one needs to go beyond the traditional semiclassical treatment

- Any particle can be put in a vortex state
  - Twisted states in heavy ion collision may be ubiquitous
  - New interpretation

- New opportunities in material science molecule physics, nuclear physics, hadron structure

physics with vortex particles is emerging.

Thank you