Study of Casimir Effect in Wigner Function Formalism

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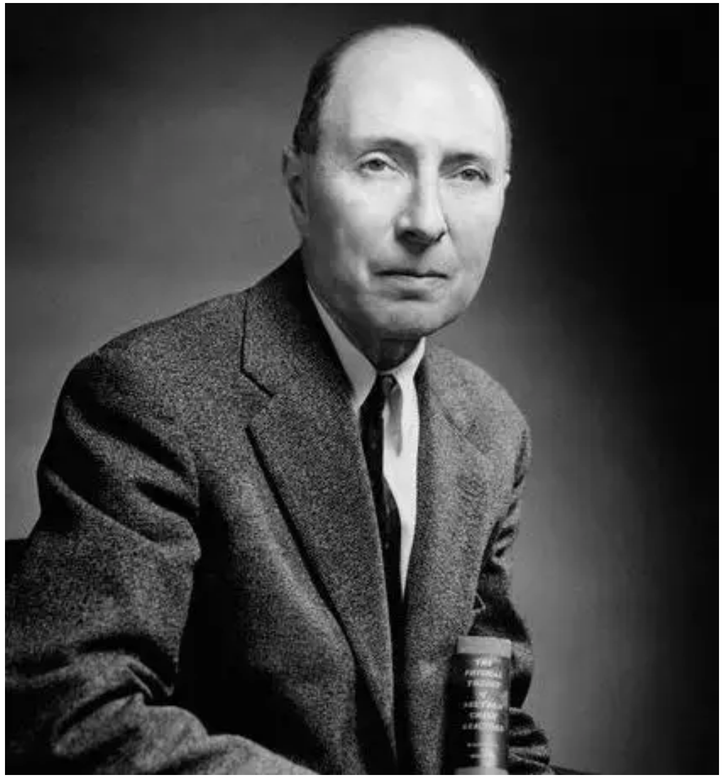
Wigner Function Formalism

- Spin degree of freedom
- Vorticity and EM field contribution
- Quantum correlation: CME, CVE, spin polarization, Berry phase, etc.
- Non-equilibrium evolution



- Used in many fields

Quantum Kinetic Theory

- Previously, mostly focused on fermions.
- Semi-classical expansion.
- Infinite size system.

- What (complete) quantum effect does a finite-size photon system have?

Casimir Effect

- Effect of the quantum vacuum
- Finite size system
- Many recent advances: repulsive, lateral, torque, non-equilibrium, …
- Experimental observations since 1997

Wigner Function

- The Wigner function for electromagnetic field

$$G_{\mu\nu}(x,p) = \int d^4y e^{-ip\cdot y} \left\langle A_\mu\left(x+\frac{y}{2}\right) A_\nu\left(x-\frac{y}{2}\right)\right\rangle$$

- Kinetic equation

$$\bar{p}_\sigma \bar{p}^\sigma A_\mu - \bar{p}_\mu \bar{p}^\sigma A_\sigma = 0$$

$$\bar{p}_\mu = p_\mu + i\partial_\mu/2$$

- Gauge fixing (Lorenz gauge)

$$\bar{p}^\mu G_{\mu\nu} = 0$$

Quantum Kinetic Equation

- Transport equation

$$p^\mu \partial_\mu G^{\pm}_{\mu\nu} = 0$$

- Constraint equation

$$\left(p^2 - \partial^2/4\right) G^{\pm}_{\mu\nu} = 0$$

- Gauge fixing equation

$$p^\mu G^{\pm}_{\mu\nu} + i\partial^\mu/2 G^{\mp}_{\mu\nu} = 0$$

$$G^{\pm}_{\mu\nu} = (G_{\mu\nu} \pm G_{\nu\mu})/2$$

- $G^{+}_{\mu\nu}$ real, $G^{-}_{\mu\nu}$ pure imaginary

Factorization

$$G^{\pm}_{\mu\nu}(x,p) = C^{\pm}_{\mu\nu}(p) f_{\pm}(x,p)\delta(p^2)$$

- On shell
- Polarization depends only on momentum
- Scalar-like real distribution function

$$C^{+}_{\mu\nu} = \frac{p_\mu p_\nu}{(p\cdot u)^2} - \frac{p_\mu u_\nu + p_\nu u_\mu}{p\cdot u} + g_{\mu\nu}$$

$$C^{-}_{\mu\nu} = i\epsilon_{\mu\nu\sigma\rho}\frac{p^\sigma u^\rho}{2p\cdot u}$$

$$u\cdot G_{\mu\nu} = 0$$

Kinetic Equations

$$p \cdot \partial f_{\pm} = 0$$

$$\partial^2 f_{\pm} = 0$$

$$\left(p_\mu u \cdot \partial - p \cdot u \partial_\mu\right) f_+ = 0$$

$$\epsilon_{\mu\nu\sigma\rho} \partial^\nu p^\sigma u^\rho f_- = 0$$

- Currents

$$j_\mu = \partial_\mu f_+ \longrightarrow \text{Number density}$$

$$j_5^\mu = p^\mu f_- \longrightarrow \text{Chiral imbalance}$$

- Energy-momentum tensor

$$t_{\mu\nu} = 2 p_\mu p_\nu f_+$$

Finite Size Effect

- Two infinite plates at $z = 0$ and $z = a$.
- $p_z = \dfrac{n\pi}{a}$
- Energy per unit area between the plates

$$\mathcal{E} = \int \frac{d^2 p_\perp}{(2\pi)^2} \sum_n \sqrt{p_\perp^2 + \left(\frac{n\pi}{a}\right)^2} f_+(p)$$

- Subtract continuous limit

$$\Delta\mathcal{E} = \int \frac{d^2 p_\perp}{(2\pi)^2} \left(\sum_n - \int dn\right) \sqrt{p_\perp^2 + \left(\frac{n\pi}{a}\right)^2} f_+(p)$$

Finite Temperature

- $f_+ = \frac{1}{e^{\epsilon_p/T} - 1} + \frac{1}{2}$
- Casimir Force

$$F_T = -\frac{\partial \Delta \mathcal{E}}{\partial a}$$

$$= \underline{\frac{\pi^2}{15}T^4} - \underline{\frac{\pi^2}{a^4}\sum_n \frac{n^3}{e^{\frac{n\pi}{aT}} - 1}} - \underline{\frac{\pi^2}{240a^4}}$$

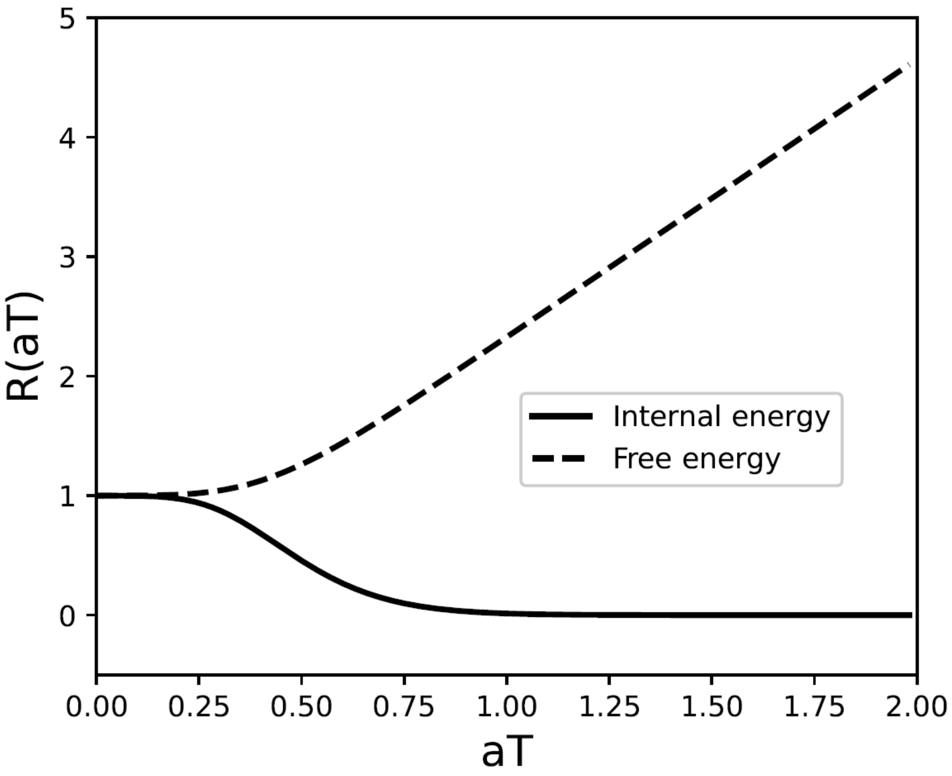
Continuous limit　　Finite T effect　　Vacuum contribution

Finite Temperature

- $R(aT) = F_T / F_0$
- Free energy and internal energy

$$\mathcal{E} = \mathcal{F} + \beta \frac{\partial \mathcal{F}}{\partial \beta}$$

- Fully thermalized vs. adiabatic.
- Free energy only applies to equilibrium states.



Non-equilibrium States

- Arbitrary initial distribution $f_0(t_0, \vec{x}, p)$
- Free-streaming solution

$$f_+ = f_0\left(t_0, \vec{x} - \frac{\vec{p}_0}{\epsilon_p}\left(t - t_0\right), p\right)$$

- Without collision, each photon propagates freely

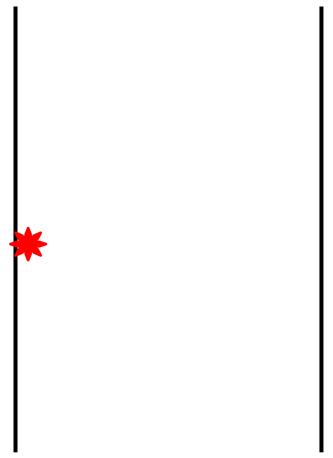
Non-equilibrium States

- Choose

$$f_0 = e^{-\epsilon_p |\vec{x}|}$$

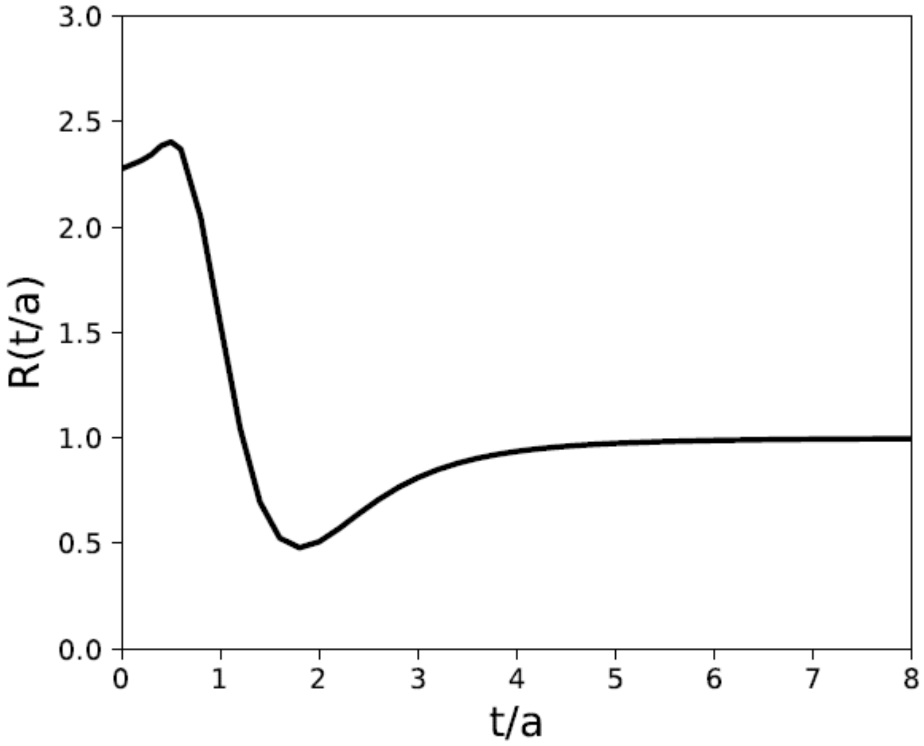
$$f_+\left(t, \vec{x}, p\right) = e^{-|\epsilon_p \vec{x} - \vec{p}\, t|}$$

- Light source at $\vec{x} = 0$, turned off at $t = 0$.



Non-equilibrium States

- Casimir force is now coordinate dependent
- We calculate the ratio at $\vec{x}_\perp = 0$
- Strong enhancement, with oscillation, then saturates quickly



Conclusion

- We studied the Casimir effect in the medium in the frame of quantum kinetic theory.
- When the system is adiabatic, the Casimir force is suppressed by increasing temperature.
- In non-equilibrium the force oscillates and decays.

Outlook

- Different non-equilibrium set-ups.
- Possible measurements.

Thank you!