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Thermalization and prethermalization

in the soft-wall AdS/QCD model

报告人：操宣敏

合作者：巢静宜（江西师范大学），Matteo Baggioli(上海交通大学),

刘绘（暨南大学），李丹凝（暨南大学）

[1] X. Cao, S. Qiu, H. Liu, and D. Li, J. High Energ. Phys. 2021, 5 (2021)
[2] X. Cao, J. Chao, H. Liu, and D. Li, [arXiv 2204.11604]
[3] X. Cao, M. Baggioli, H. Liu, and D. Li, Pions dynamics in a soft AdS-QCD model, in submission

第一届“粤港澳”核物理论坛

2022/07/5

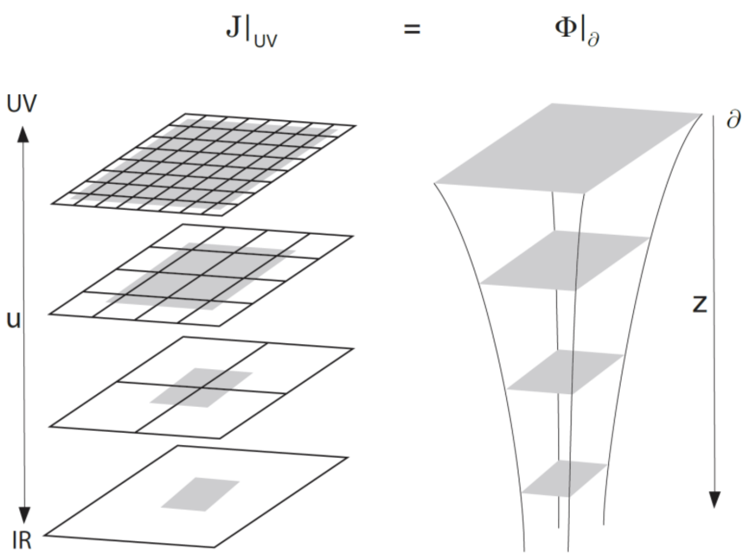
提纲

1. 研究动机

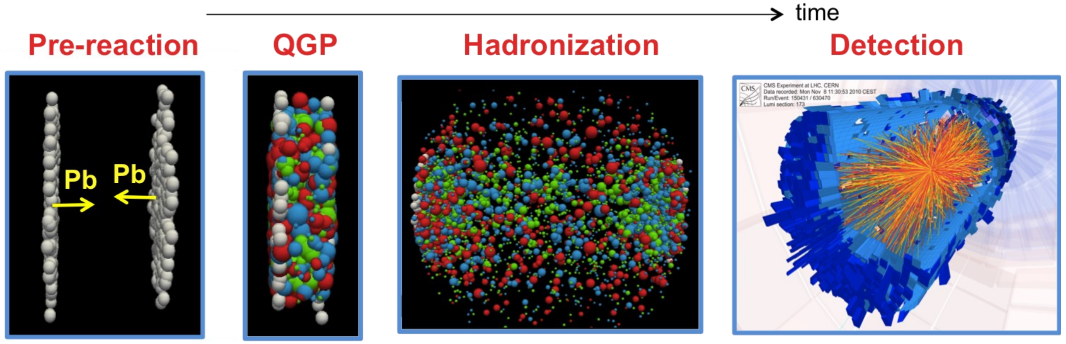
2. 软墙模型及背景介绍

3. 热化与预热化过程

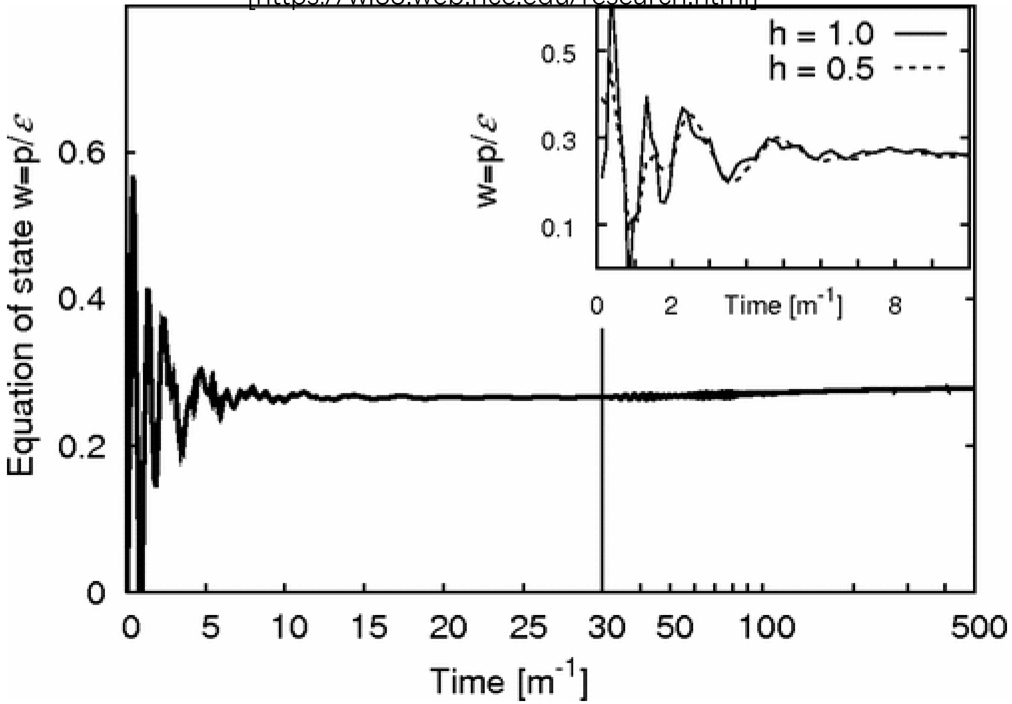
4. 总结



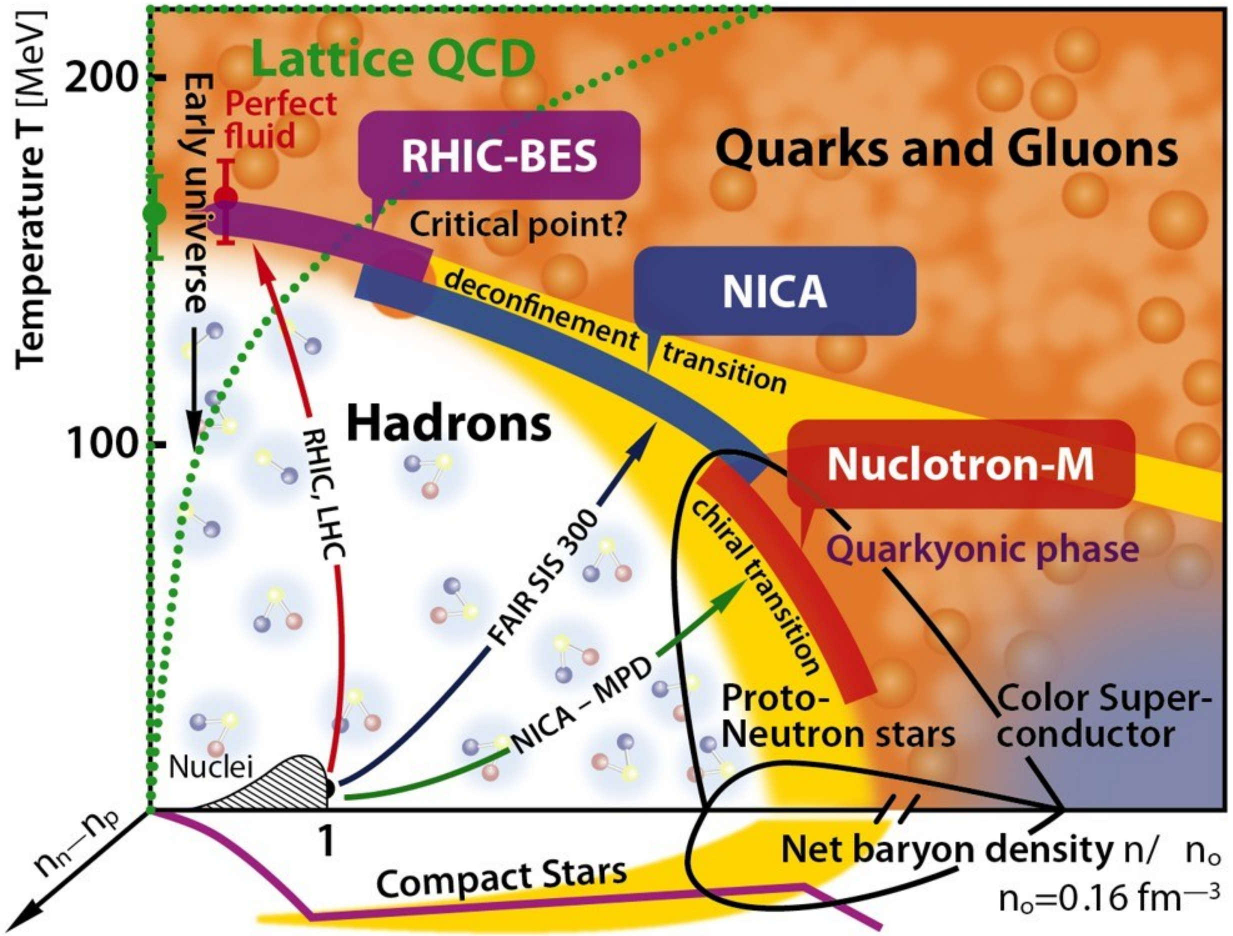
研究动机



[https://wl33.web.rice.edu/research.html]



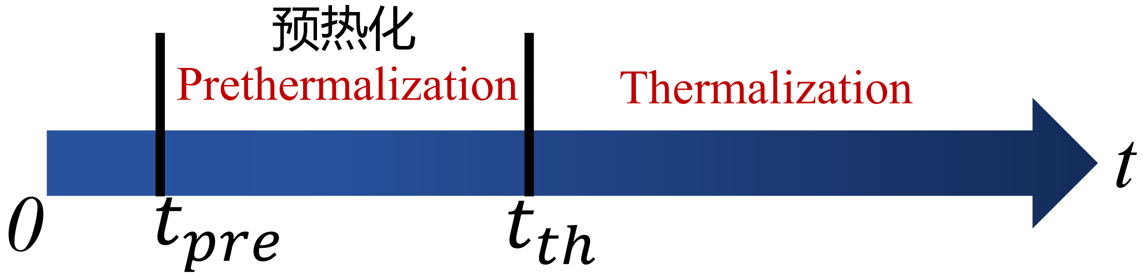
[J. Berges et al. PhysRevLett.93.142002]



[NICA physics at JINR (Official Web-Page). nica.jinr.ru/physics.php]

The ratio of pressure over energy density w as a function of time. The inset shows the early stages for two different couplings and demonstrates that the prethermalization time is independent of the interaction details.

研究动机



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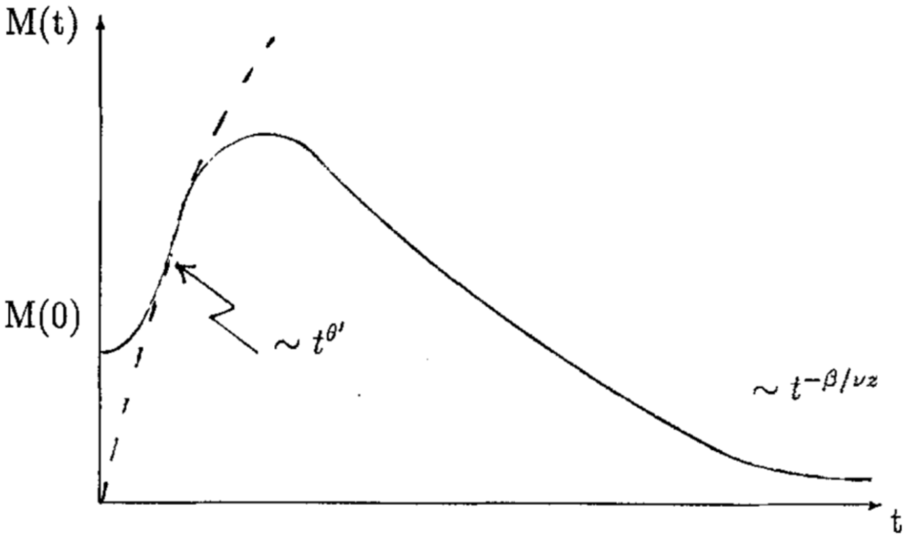
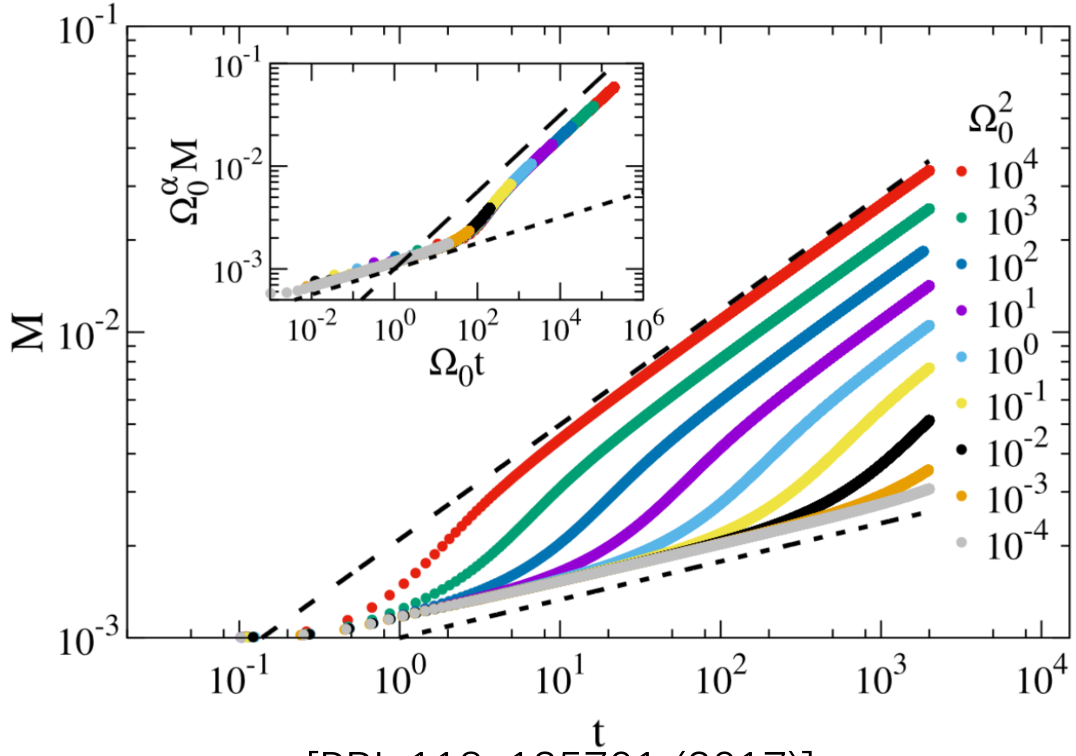
短时动力学（经典 Ising模型）

(初始临界滑移)



[H.K. Janssen et al. Z. Phys.B-CondensedMatter 73, 539-549(1989)]

预热化动力学（量子系统）



[PRL 118, 135701 (2017)]

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软墙AdS/QCD模型（$N_f = 2$）

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[PhysRevD.74.015005, PhysRevLett.95.261602]

$$S = \int d^5x \sqrt{g} e^{-\Phi(z)} \mathrm{Tr}\left\{ |D_M X|^2 - V(|X|) - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right) \right\}$$

$$X = (\chi + S) t^0 e^{-i2\pi^a t^a}, t^0 = \frac{I_2}{2}, t^a = \sigma^i/2$$

可以同时实现Regge Trajectories和手征对称自发破缺。【*Physics Letters B* **762**, 86–95 (2016).】

关于χ的EOM：$\chi'' + \left(3A' + \frac{f'}{f} - \Phi'\right)\chi' + \frac{e^{2A}}{f}\left(m_5^2 - \frac{\lambda\chi^2}{2}\right)\chi = 0$

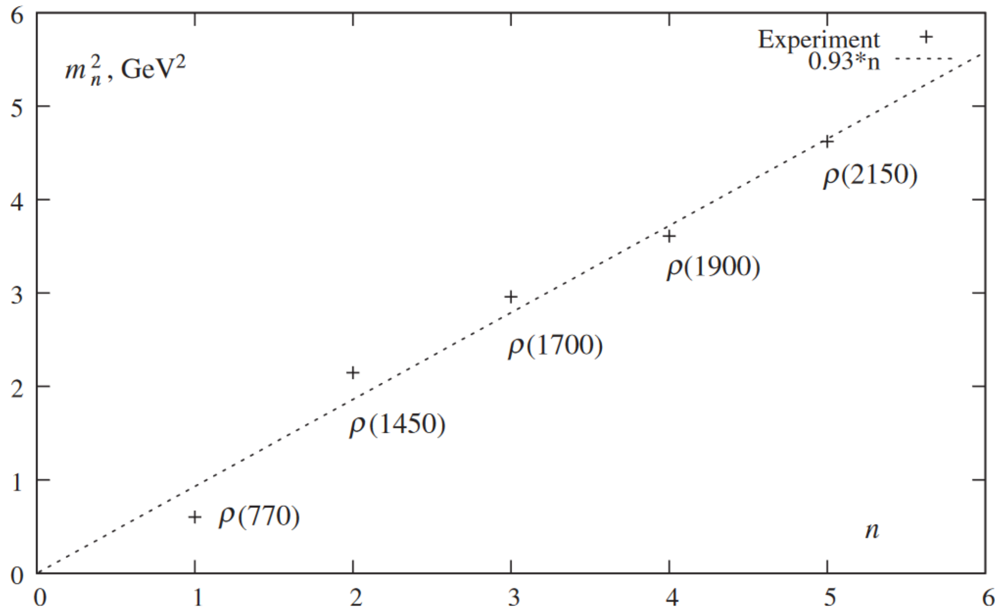
在UV边界的渐进展开解：

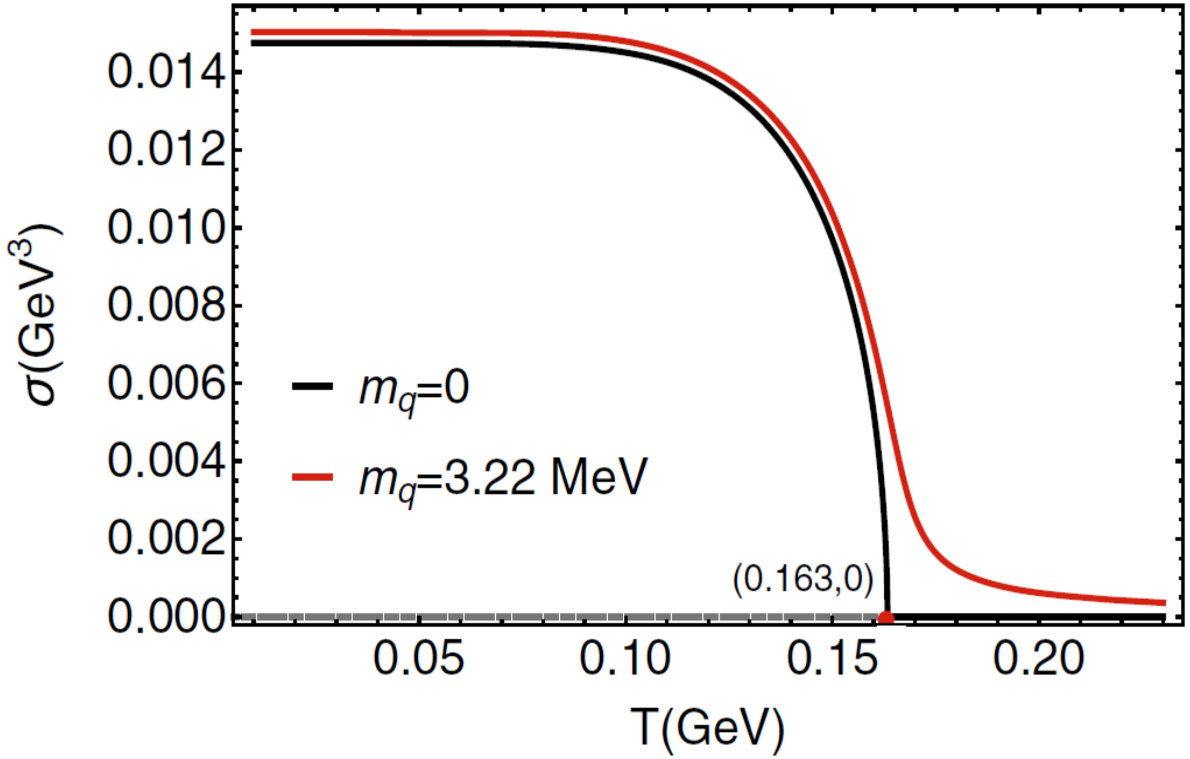
$$\chi(z \to 0) = m_q \zeta z + \frac{\sigma}{\zeta} z^3 + \cdots$$

$\langle \bar{q} q \rangle$手征凝聚值

$m_u = m_d = m_q$夸克质量

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6 ± 0.0004 [8]	139.6^*	141
m_ρ	775.8 ± 0.5 [8]	775.8^*	832
m_{a_1}	1230 ± 40 [8]	1363	1220
f_π	92.4 ± 0.35 [8]	92.4^*	84.0
$F_\rho^{1/2}$	345 ± 8 [15]	329	353
$F_{a_1}^{1/2}$	433 ± 13 [6]	486	440
$g_{\rho\pi\pi}$	6.03 ± 0.07 [8]	4.48	5.29





两点关联函数-$G_R(\omega,p)$

取规范（赝标量与轴矢量脱耦）$a_\mu^i = a_\mu^{T,i} + \partial_\mu \varphi^i, \partial^\mu a_\mu^{T,i} = 0$

$$S = \frac{1}{2g_5^2}\int d^5x\sqrt{g}\, e^{-\Phi}\Big\{ g_5^2\Big[g^{\mu\nu}\partial_\mu S\partial_\nu S + g^{zz}(\partial_z S)^2 - m_5^2 S^2 - \frac{3\lambda}{2}\chi^2 S^2\Big]$$
$$- \sum_{i=1}^{3}\Big\{g^{\mu\nu}g^{zz}\partial_z\partial_\mu\varphi^i\partial_z\partial_\nu\varphi^i - g_5^2\chi^2\Big(g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^i + g^{\mu\nu}\partial_\mu\pi^i\partial_\nu\pi^i + g^{zz}\left(\partial_z\pi^i\right)^2 - 2g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\pi^i\Big)\Big\}\Big\}$$

运动方程: $S'' + \left(3A' + \frac{f'}{f} - \Phi'\right)S' + \left(\frac{\omega^2 - p^2 f}{f^2} - \frac{2m_5^2 + 3\lambda\chi^2}{2f}A'^2\right)S = 0$, $\left(f = 1 - \frac{z^4}{z_h^4}\right)$

在$z = 0$处的渐进展开解：

对偶到标量介子场算符

$$S(x \to 0) = s_1 z + s_3 z^3 - \frac{1}{4}s_1\left[2\left(\omega^2 - p^2 + \mu_c^2 - 2\mu_g^2\right) - 3\zeta^2\lambda m_q^2\right]z^3\log(z) + \cdots$$

[DT Son et al. *JHEP* **2002**, 042–042 (2002).]

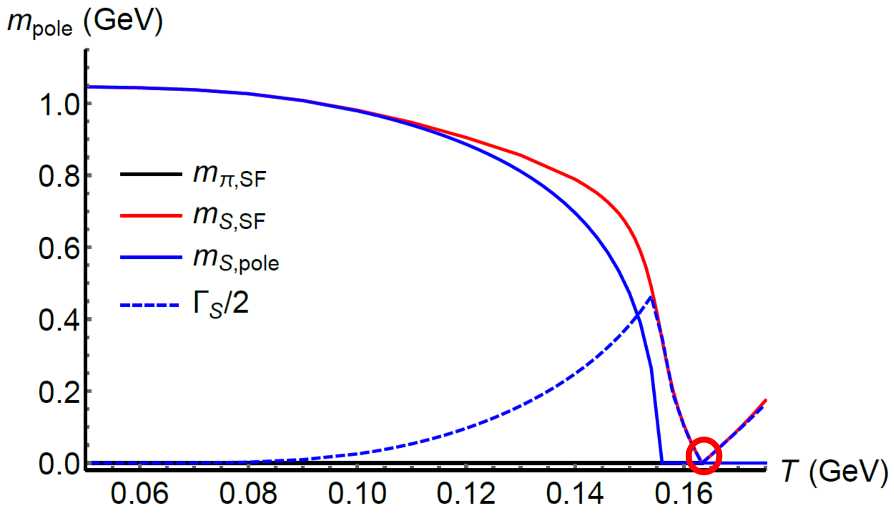
对偶到外源 J_s

$$G_S^R(\omega,p) = \frac{\delta^2 S_s^{on}}{\delta J_s^*\delta J_s}\Big|_{z=\epsilon} = -4\frac{s_3}{s_1} - \frac{3}{4}\zeta^2\lambda m_q^2 + \frac{1}{2}\left(-2\mu_g^2 + \mu_c^2 + \omega^2 - p^2\right)$$

标量、π介子的有效质量

➤谱函数质量（谱函数第一个峰的位置）

● $\omega \neq 0$ 且 $p = 0$　$\rho(\omega) = -\frac{1}{\pi}\mathrm{Im}[G^R(\omega)]$



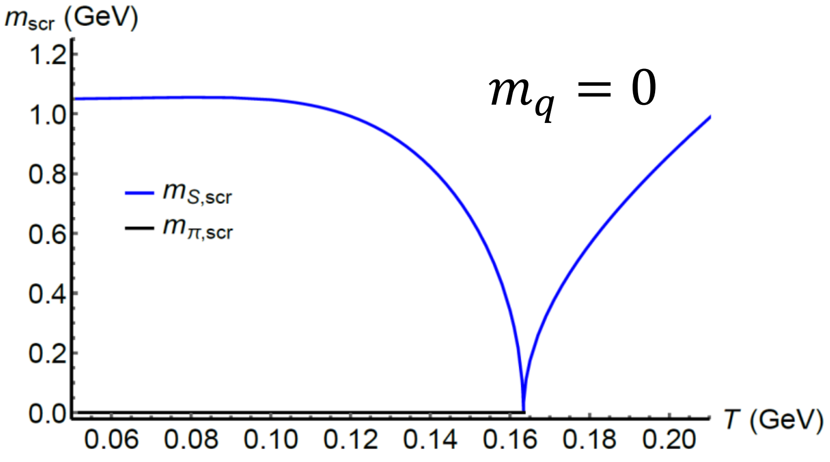
● 临界温度：$T_c = 0.163$GeV，π介子为手征相变的Goldstone玻色子

● $m_{SF}^2 = m_{pole}^2 + (\Gamma/2)^2$　（在低温和高温区域，谱函数质量分别由pole mass和热宽度主导）

➤ Pole质量：QNM频率——ω_0

$\Rightarrow m_{pole} = \mathrm{Re}[\omega_0]$;　$\Gamma/2 = -\mathrm{Im}[\omega_0]$

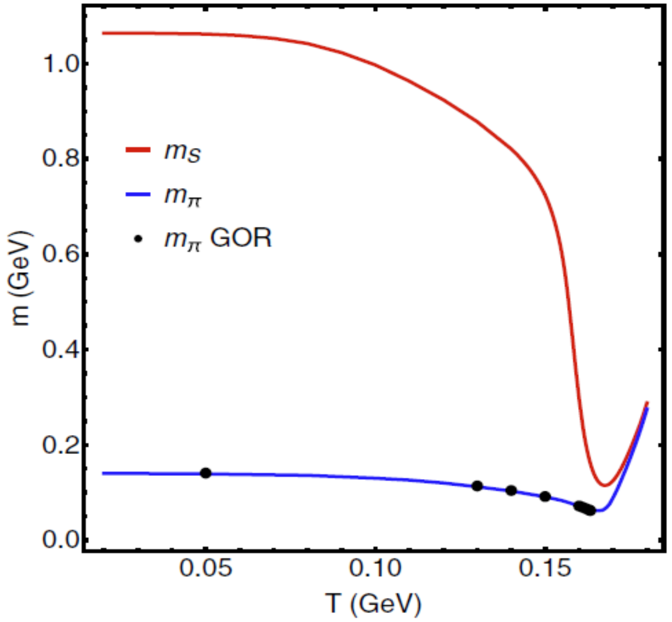
➤ 屏蔽质量 $G^R(x) \sim e^{-m_{scr}x}$

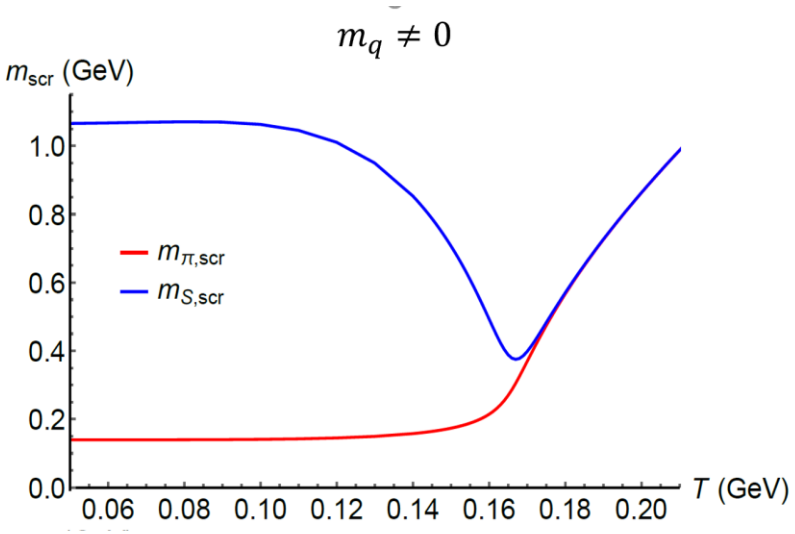


● 拟合 $\zeta = \frac{1}{\mathrm{m_{scr}}} = \left(\frac{|\mathrm{T}-\mathrm{T_c}|}{\mathrm{T_c}}\right)^{\nu}$

● 关联长度的临界指数：$\nu \approx 0.5$ (平均场)

标量、π介子的有效质量





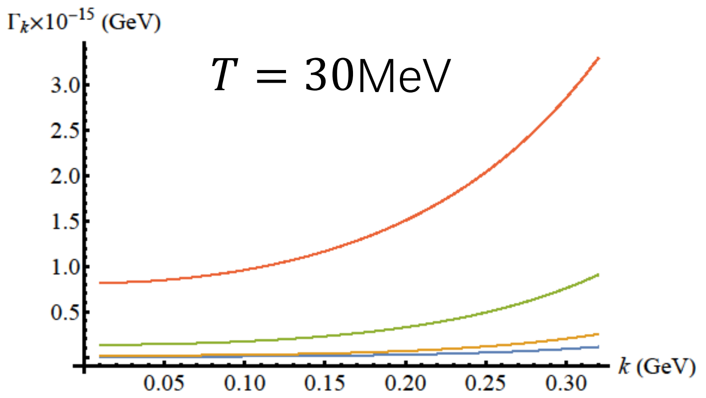
- Corssover：$T_{cp} = 0.164$GeV，π介子为pseudo-Goldstone玻色子， $m_{SF}^2 = m_{pole}^2 + (\Gamma/2)^2$
- T_{cp}附近π介子质量下降约60%，则其在末态低动量区产额可能有大幅增加
- GOR关系: $m_\pi^2 f_{\pi,T}^2 = 2m_q\sigma$

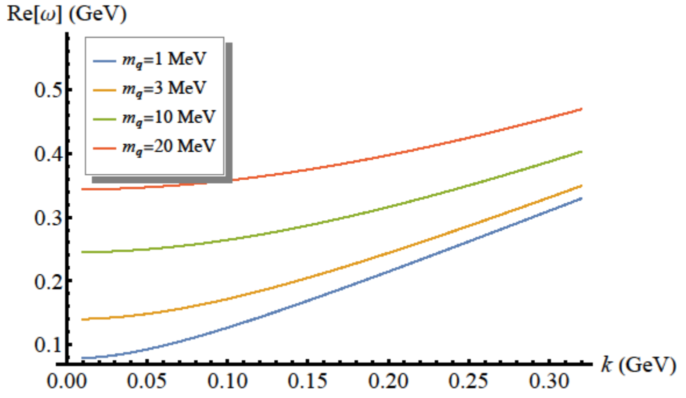
π介子色散关联

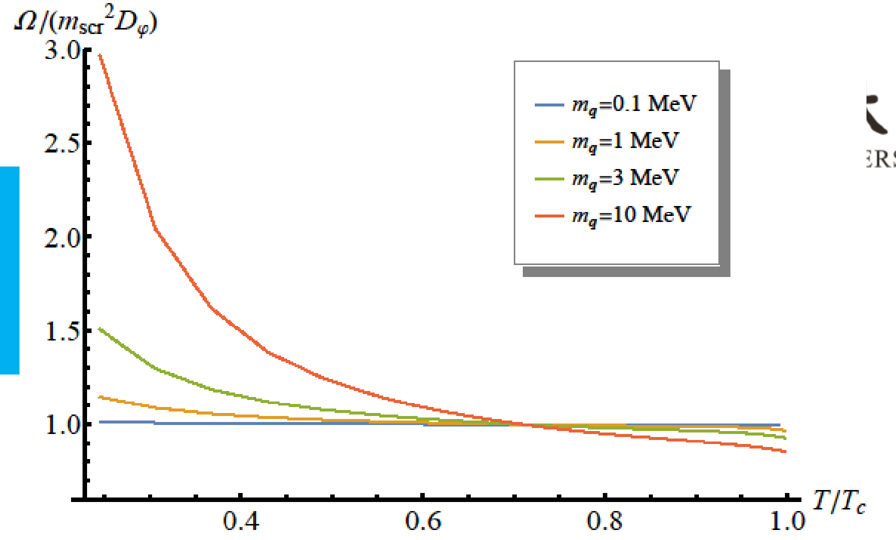
$$\omega = \pm vk - \frac{i}{2}\mathrm{D}\, k^2 + \cdots \quad (\mathrm{D} = D_\varphi + D_5)$$

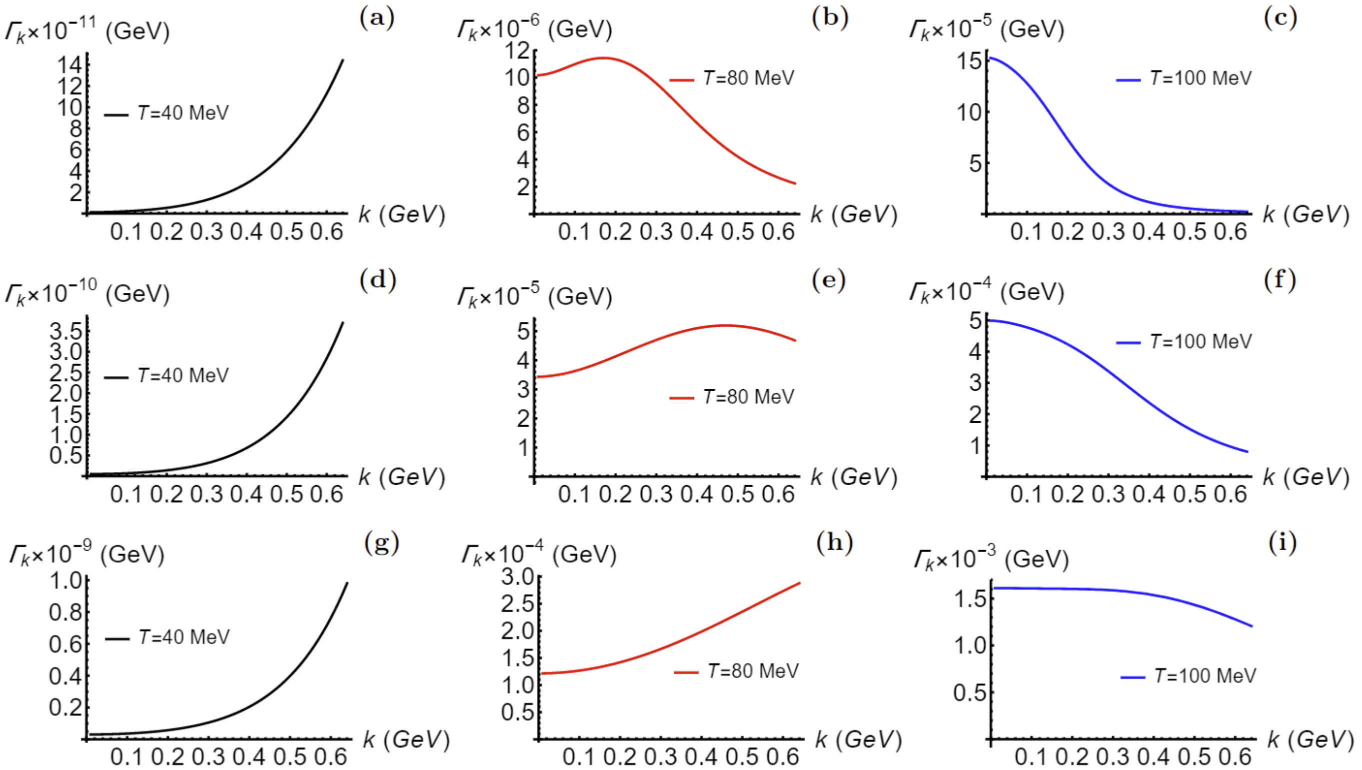
$$\Gamma_k = D_\varphi m_{scr}^2 + (D_5 + D_\varphi)k^2 + \cdots$$

$$\Gamma_k(k=0) = \Omega = D_\varphi m_{scr}^2$$









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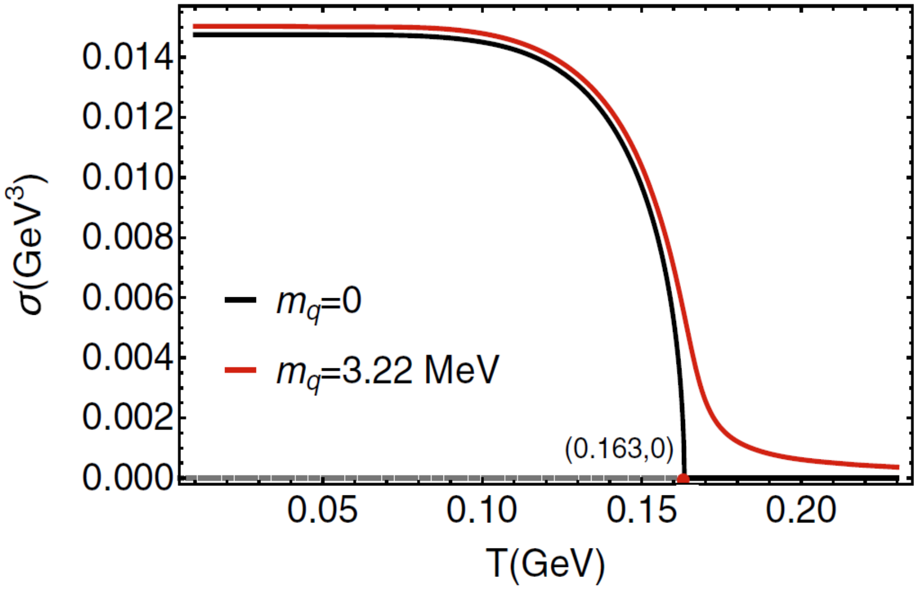
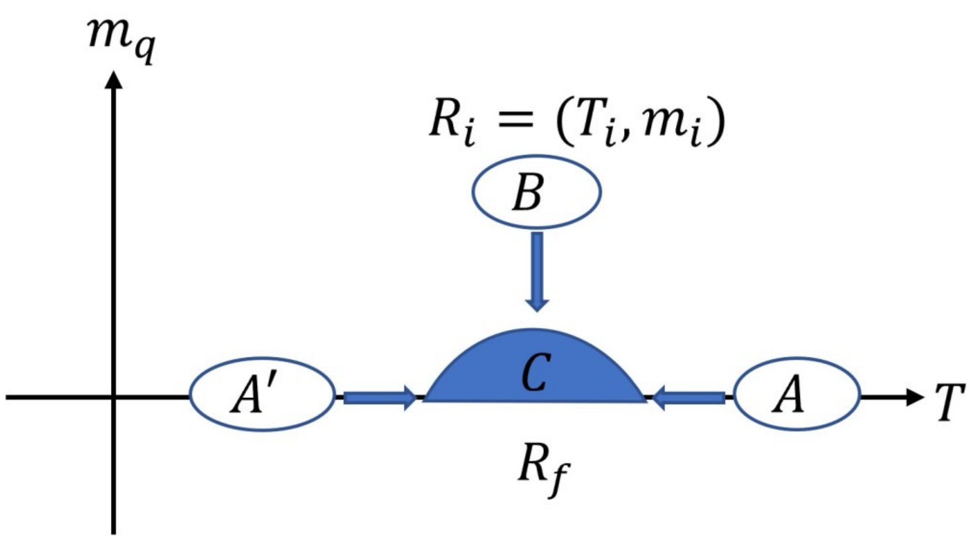
非平衡演化方案（Sudden quench）



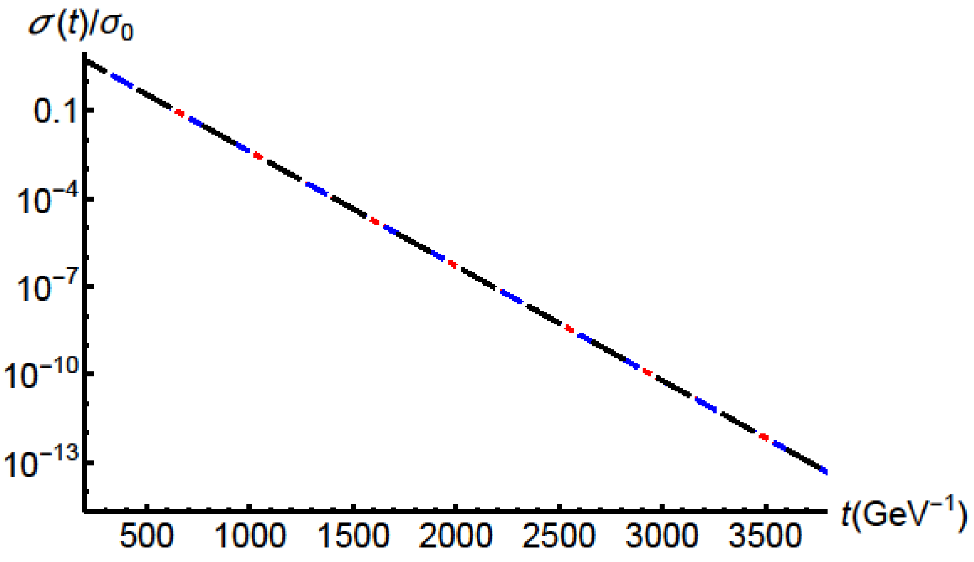


弛豫过程

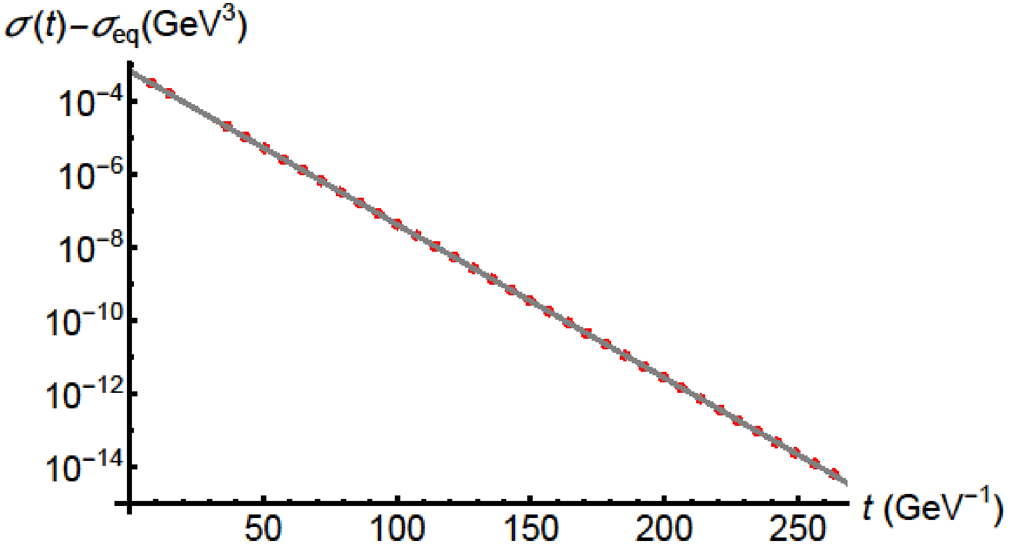
在线性响应理论框架下，序参量应满足：

$$\frac{\partial}{\partial t}\sigma(\epsilon,t)=-\frac{\sigma(\epsilon,t)-\sigma_{eq}}{\tau_R} \quad \Rightarrow \quad \sigma(t)-\sigma_{eq}\sim e^{-(t-t_0)/\tau_R}$$

温度接近Tc



低温区域



临界点附近的标度变换

关联长度：$\xi(\epsilon, m_q, t) = b\xi(\epsilon\, b^{1/\nu}, m_q b^{\beta\delta/\nu}, t b^{-z})$

关联时间： $\tau_R(\epsilon, m_q, t) = b^z \tau_R(\epsilon\, b^{1/\nu}, m_q b^{\beta\delta/\nu}, t b^{-z})$

手征凝聚：$\sigma(\epsilon, m_q, t) = b^{-\beta/\nu}\sigma(\epsilon\, b^{1/\nu}, m_q b^{\beta\delta/\nu}, t b^{-z})$

领头阶行为：

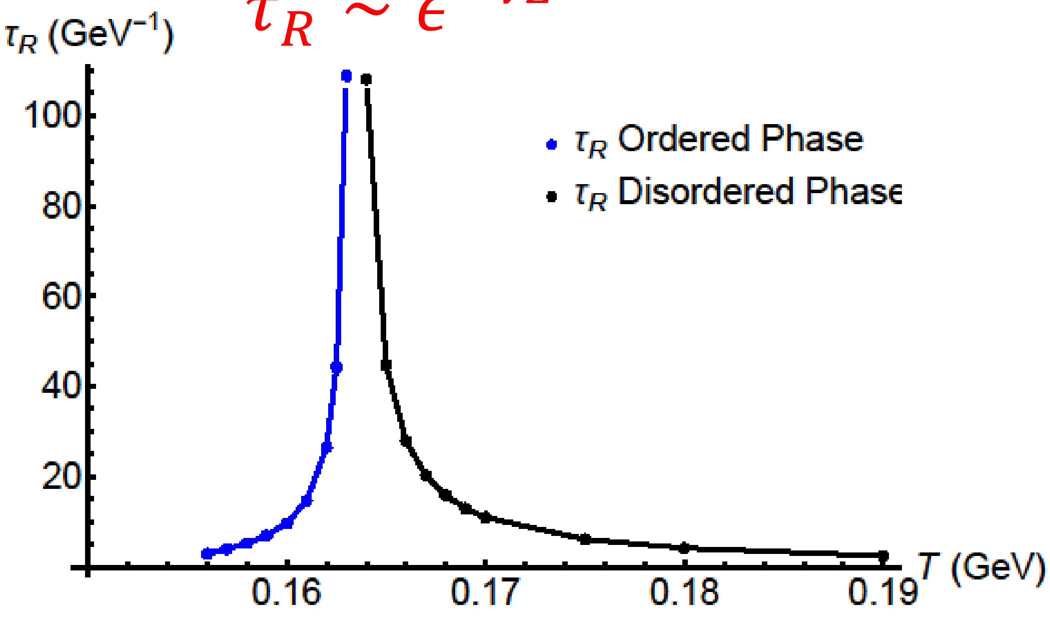
$$\xi \sim m_q^{-\nu/\beta\delta} \qquad \sigma \sim m_q^{1/\delta} \qquad \tau_R \sim m_q^{-\nu z/\beta\delta}$$

$$\xi \sim \epsilon^{-\nu} \qquad \sigma \sim \epsilon^{\beta} \qquad \tau_R \sim \epsilon^{-\nu z}$$

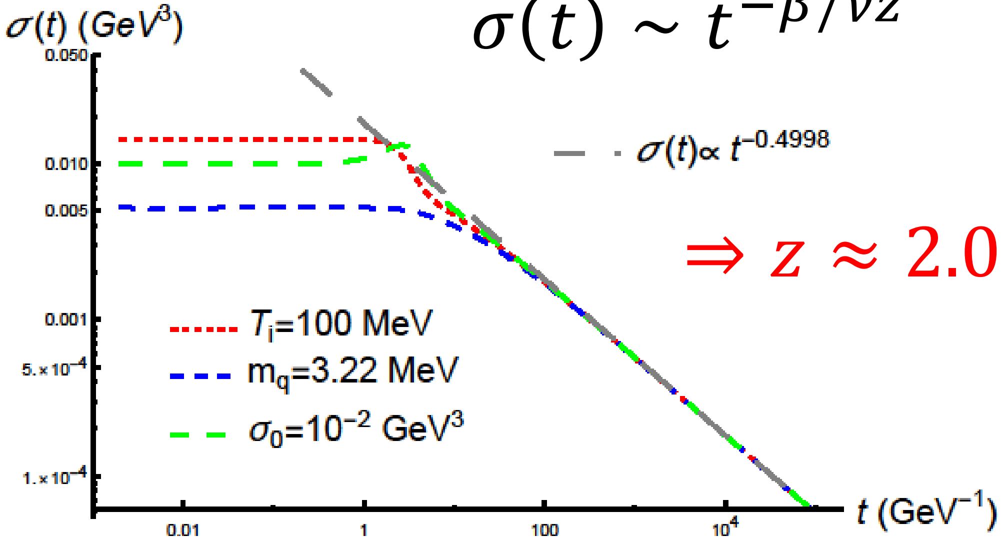
临界慢化：$\sigma(t) \sim t^{-\beta/\nu z}$

临界慢化（$\beta = 1/2, \delta = 3, \nu = 1/2$）

$\tau_R \sim \epsilon^{-\nu z}$



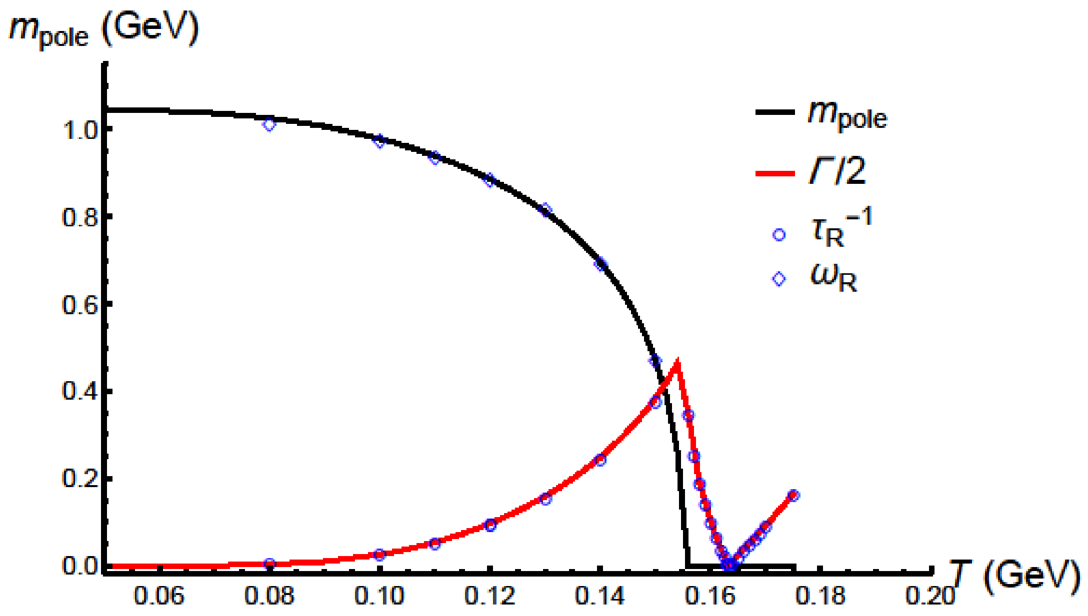
$\sigma(t) \sim t^{-\beta/\nu z}$

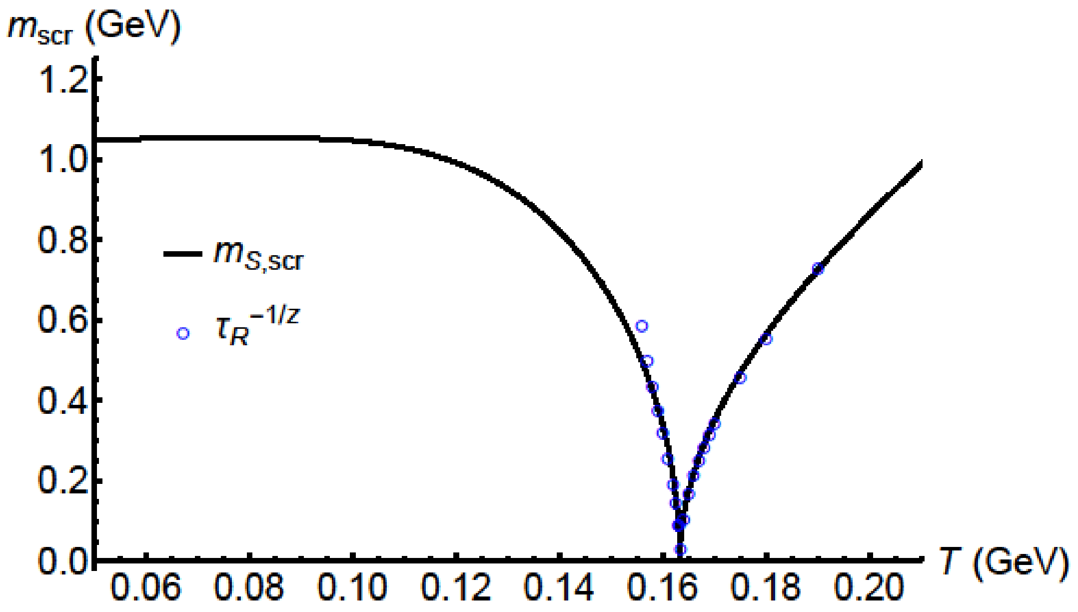


$\Rightarrow z \approx 2.0008$

动力学指数：$z \approx 2.08$ 或 2.04
(属于平均场普适类)

关联时间、长度与QNM的关系



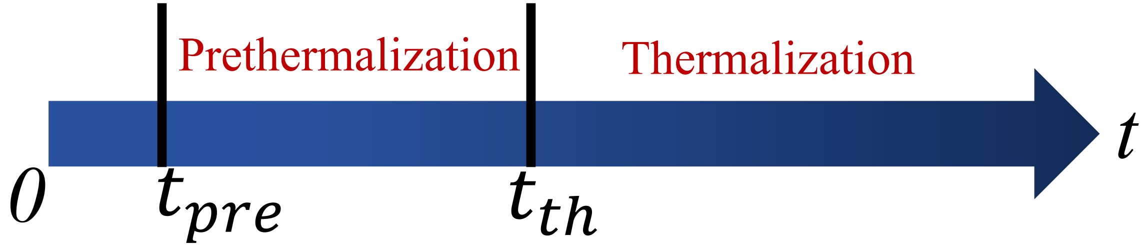


$\tau_R = \Gamma/2$, $m_{pole} \sim \sigma$演化的振荡频率， $\xi = m_{scr}^{-1} \sim \tau_R^{-1/z}$

预热化（Prethermalization）

关联长度：$\xi(R_i,\epsilon,m_q,t)$

$= b\xi(R_i(b),\epsilon\, b^{1/\nu},m_q b^{\beta\delta/\nu},tb^{-z})$



手征凝聚：$\sigma(R_i,\epsilon,m_q,t) = b^{-\beta/\nu}\sigma(R_i(b),\epsilon\, b^{1/\nu},m_q b^{\beta\delta/\nu},tb^{-z})$

$$\{m_i\, b^{x\beta\delta/\nu},\ \epsilon_i b^{x/\nu},\ \sigma_i\, b^{x\beta/\nu}\}$$

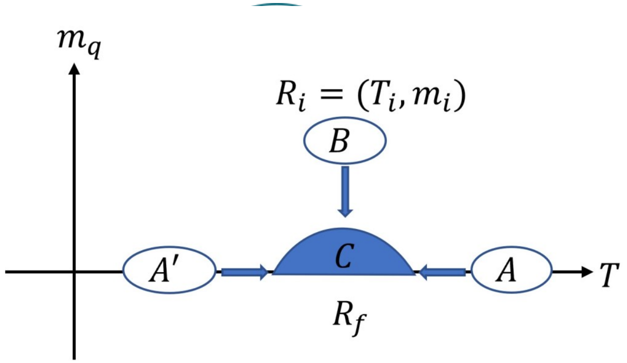
初态$\chi(T\gg T_c)=\frac{\sigma_i}{\gamma}z^3$，Quench 到临界点$\{\epsilon,m_q\}=\{0,0\}$

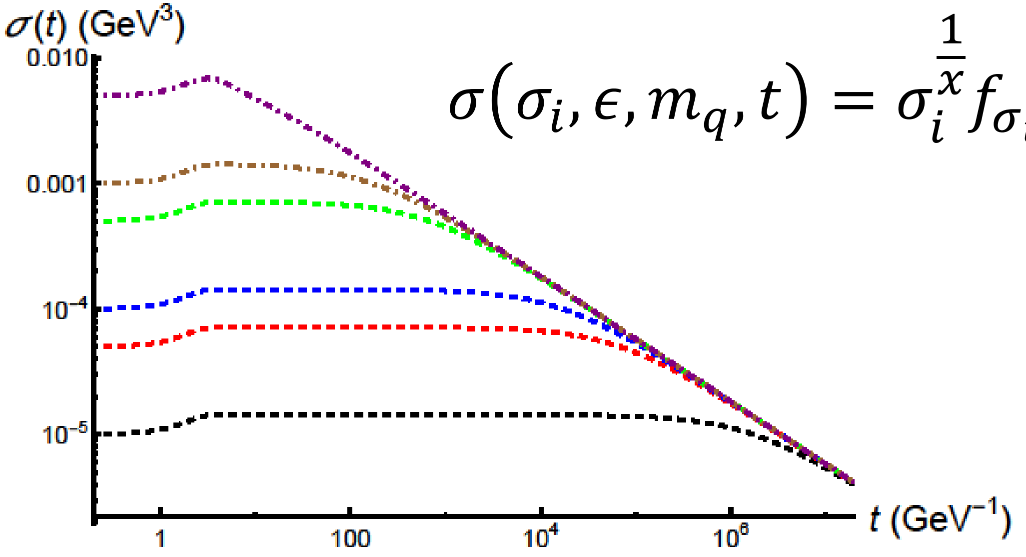
让 $tb^{-z}=\text{Const}\Rightarrow\ \sigma(\sigma_i,t)=t^{-\beta/\nu z}f_t(\sigma_i t^{x\beta/\nu z})$

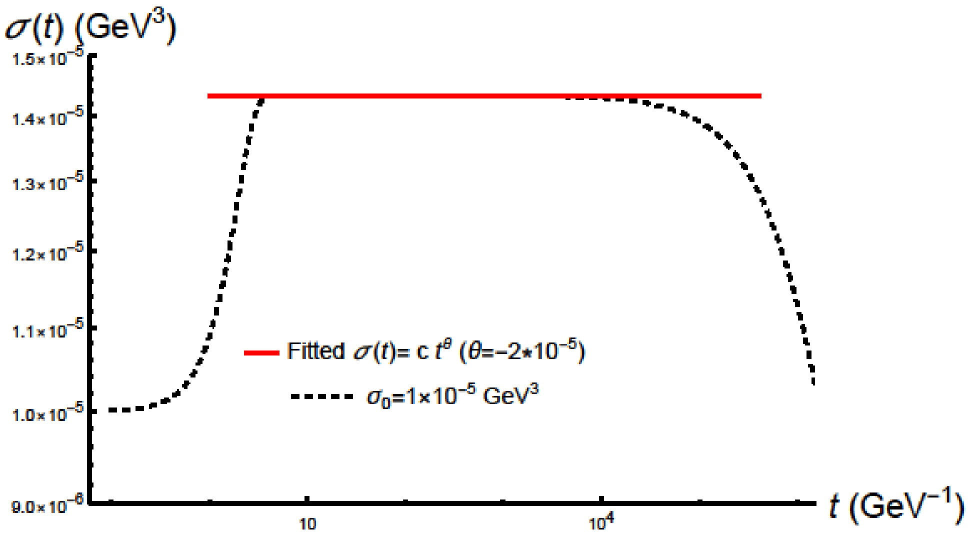
$$\begin{cases} 1、\sigma_i t^{\frac{x\beta}{\nu z}}\gg 1, & f_t=\text{Const} & \sigma(t)\propto t^{-\frac{\beta}{\nu z}} \\ 2、\sigma_i t^{x\beta/\nu z}\ll 1 \text{即}\ t\ll t_{th}\propto\sigma_i^{-\frac{\nu z}{x\beta}} & f_t\propto\sigma_i t^{\frac{x\beta}{\nu z}} & \sigma(t)\propto\sigma_i\, t^{\theta} \end{cases}$$

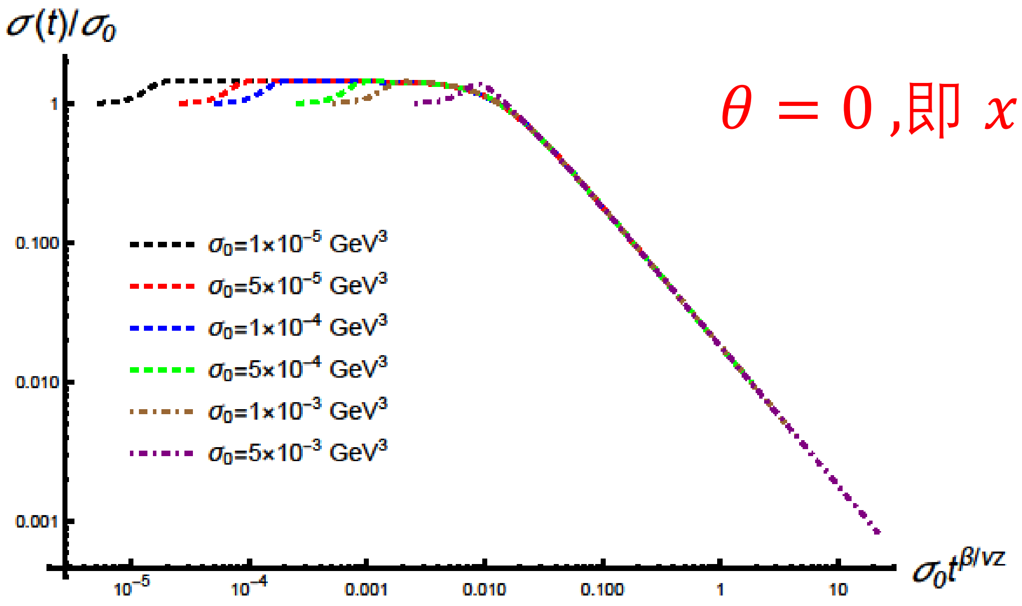
$A \rightarrow$ Critical point

$$\sigma(\sigma_i, \epsilon, m_q, t) = \sigma_i^{\frac{1}{x}} f_{\sigma_i}\left(\cancel{\epsilon \sigma_i}^{-\frac{1}{x\beta}}, \cancel{m_q}\sigma_i^{-\frac{\delta}{x}}, t\sigma_i^{\frac{\nu z}{x\beta}}\right)$$

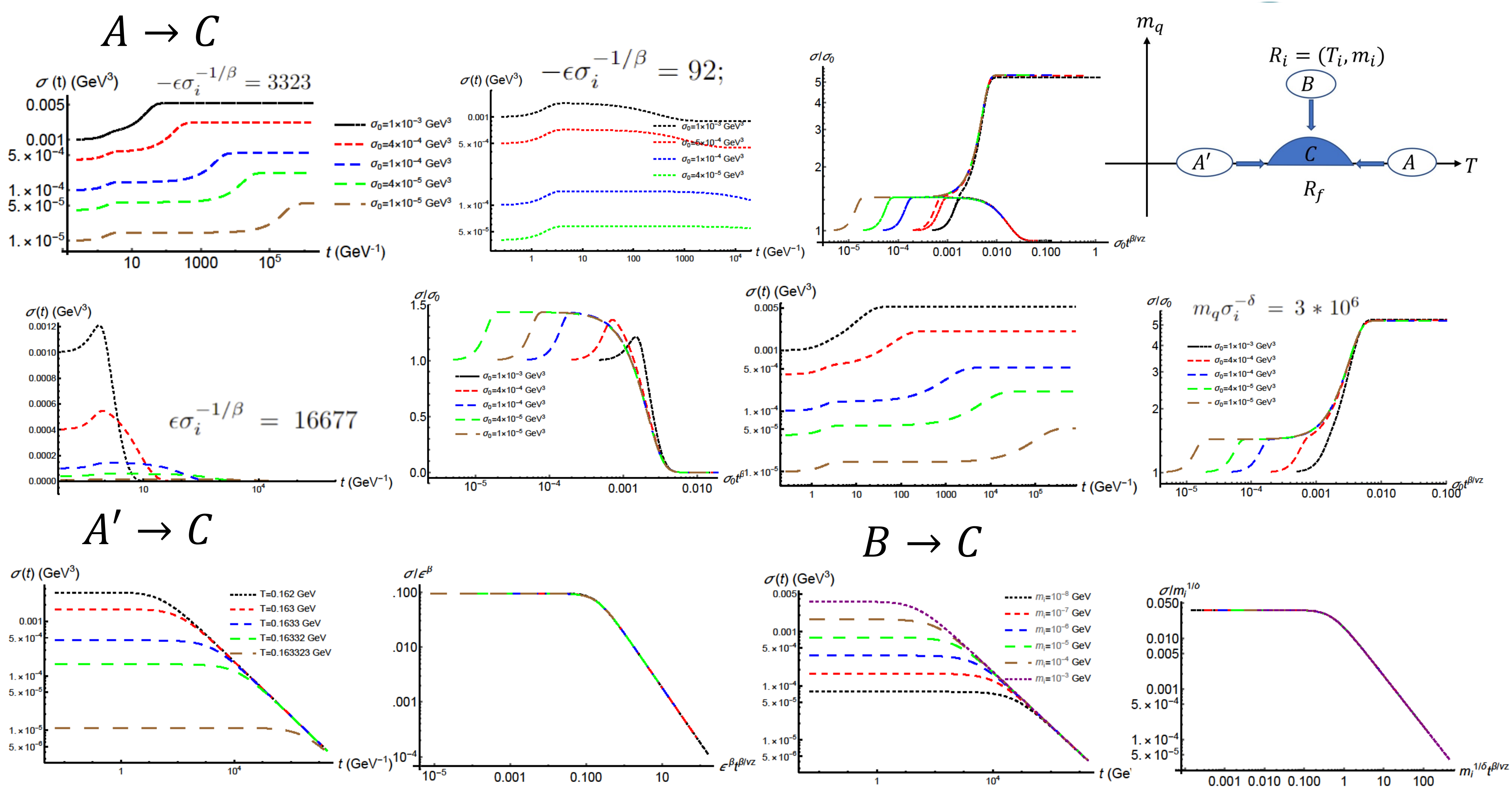








$\theta = 0$,即 $x = 1$


A → C
A′ → C
B → C
$-\epsilon\sigma_i^{-1/\beta} = 3323$
$-\epsilon\sigma_i^{-1/\beta} = 92;$
$\epsilon\sigma_i^{-1/\beta} = 16677$
$m_q\sigma_i^{-\delta} = 3 * 10^6$
m_q
$R_i = (T_i, m_i)$
B
C
A′
A
T
R_f
σ (t) (GeV³)
t (GeV⁻¹)
σ/σ₀
$\sigma_0 t^{\beta/\nu z}$
σ₀=1×10⁻³ GeV³
σ₀=4×10⁻⁴ GeV³
σ₀=1×10⁻⁴ GeV³
σ₀=4×10⁻⁵ GeV³
σ₀=1×10⁻⁵ GeV³
T=0.162 GeV
T=0.163 GeV
T=0.1633 GeV
T=0.16332 GeV
T=0.163323 GeV
σ/ϵ^β
$\epsilon^\beta t^{\beta/\nu z}$
mᵢ=10⁻⁸ GeV
mᵢ=10⁻⁷ GeV
mᵢ=10⁻⁶ GeV
mᵢ=10⁻⁵ GeV
mᵢ=10⁻⁴ GeV
mᵢ=10⁻³ GeV
$\sigma/m_i^{1/\delta}$
$m_i^{1/\delta} t^{\beta/\nu z}$

总结

① 很好的描述了手征相变的Goldstone 玻色子性质。得到有限温度下的标量和赝标量介子的pole质量和屏蔽质量以及Goldstone玻色子的色散关系。

② 很好的描述非平衡弛豫行为，得到动力学指数$z \approx 2$

③ 在全息软墙模型中实现了短时动力学，得到初始短时动力学指数（或临界滑移指数）$\theta \approx 0$。