Nuclear matter equation of state and neutron star properties with relativistic Brueckner-Hartree-Fock theory

Chencan Wang
Supervisors: Prof. Jinniu Hu & Prof. Jiangming Yao

第一届粤港澳大湾区核物理论坛
珠海 07 03—07 06 2022

In collaboration with: Hui Tong (童辉@tjnu), Sibo Wang (王锶博@cqu)

Outline

- Introduction
- Nuclear Matter
- Neutron Star Properties
- Summary & Perspectives

Introduction

Relativistic ab initio in nuclear structure

Realistic mesonic nuclear force

Bonn potential

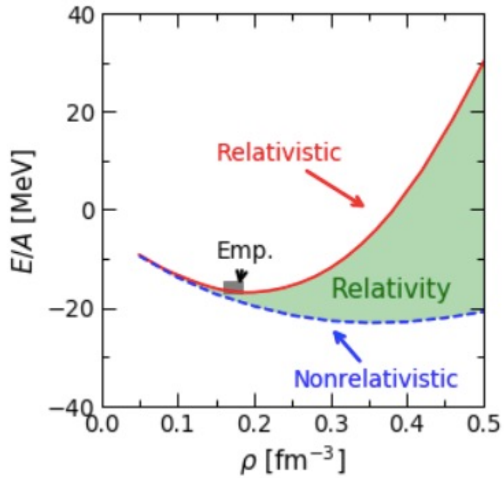
R Machleidt, et al., PhysRep149, 1 (1987), AdvNuclPhys19, 189 (1989)

Paris potential

M Lacombe, et al., PhysRevC21, 861 (1980)

Nijmegen potential…

V G J Stoks, et al., PhysRevC46, 2950 (1994)



Relativistic Brueckner-Hartree-Fock (RBHF) theory

Early proposals for RBHF

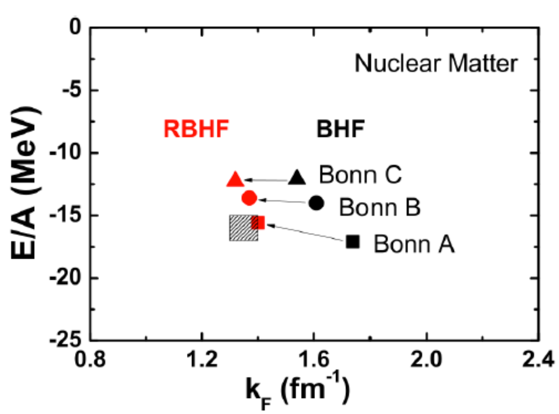
D Anastasio et al., PhysRep100, 327 (1983)

C J Horowitz & B Serot, NuclPhysA464, 613 (1987)

Nuclear matter with momentum-independence approximation (mom.-ind. app.)

R Brockmann & R Machleidt, PhysRevC42, 1965 (1990)

G Q Li, et al., PhysRevC45, 2782 (1992)



RBHF theory, cont'd

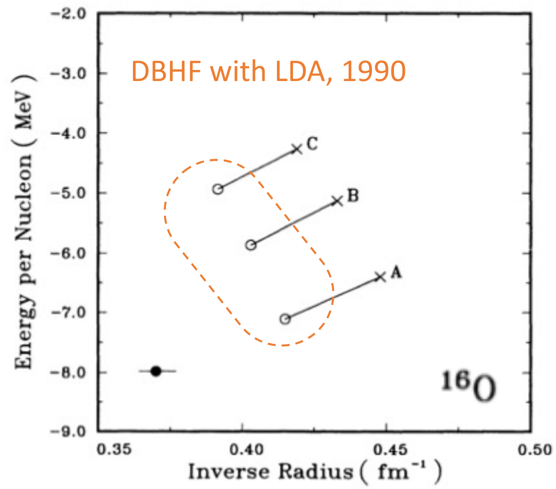
Finite nuclei with local-density approximation

H Muether, et al., PhysRevC42, 1981 (1990)

RBHF with projection method

T Gross-Boelting, et al., NuclPhysA648, 105 (1999)

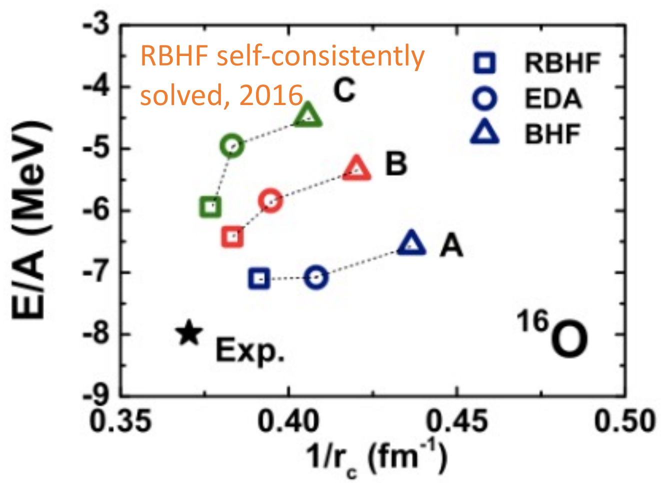
E V E Dalen, NuclPhysA744, 227 (2004)



……

Finite nuclei with fully self-consistent solutions

S H Shen, et al., ChinPhysLett33, 102103 (2016)
PhysRevC96, 014316 (2017)



New high-precision Bonn potentials & applications

C C Wang, et al., ChinPhysC43, 114107 (2019)
JPhysG47, 105108 (2020)

RBHF in full Dirac space

S B Wang, et al., PhysRevC103, 054319 (2021) *and more to be talked next…*

Why relativity

- **Nuclear matter saturation properties successfully reproduced with two-body force only**

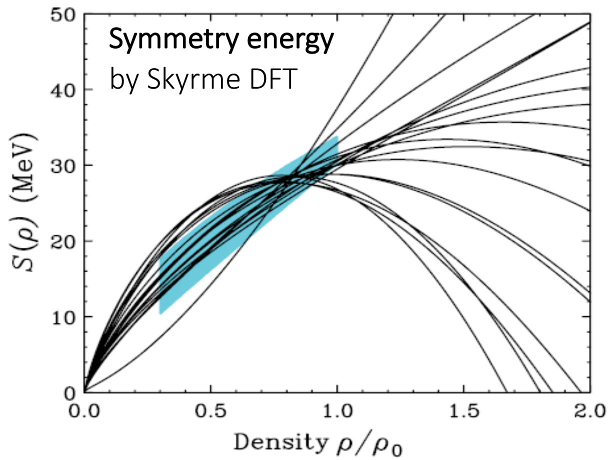
  R Brockmann & R Machleidt, PhysRevC42, 1965 (1990)

- **Some non-relativistic phenomena have relativistic origins**

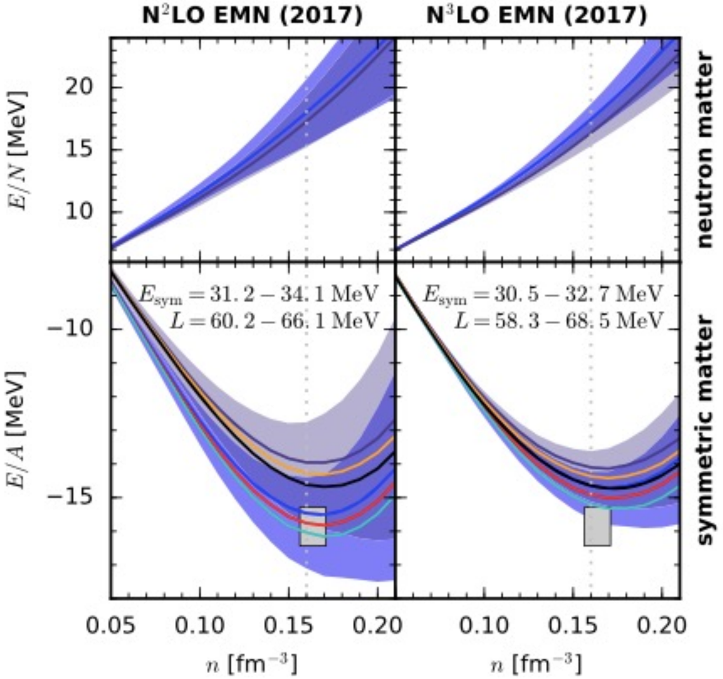
  Spin-orbital coupling & pseudospin symmetry as evidences for scalar & vector potential

  L Blokhin, et al., PhysRevLett74, 4149 (1995)

- **Non-relativistic calculations exhibit high uncertainties at high density**



M B Tsang, et al., PhysRevLett86, 015803 (2012)



C Drischler, et al., PhysRevLett122, 042501 (2019)

The nuclear matter studied with relativistic Brueckner-Hartree-Fock theory

Theoretical framework

Machleidt, et al., AdvNuclPhys19, 189 (1989)

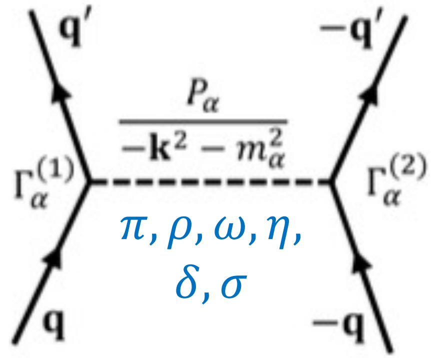
One-boson-exchange potential

$$\mathcal{L}^{(\mathrm{ps})} = -g_{\mathrm{ps}}\bar{\psi}i\gamma^5\psi\cdot\phi^{(\mathrm{ps})}$$

$$\mathcal{L}^{(\mathrm{pv})} = -\frac{f_{\mathrm{pv}}}{m_{\mathrm{pv}}}\bar{\psi}\gamma^5\gamma^\mu\psi\cdot\partial_\mu\phi^{(\mathrm{pv})}$$

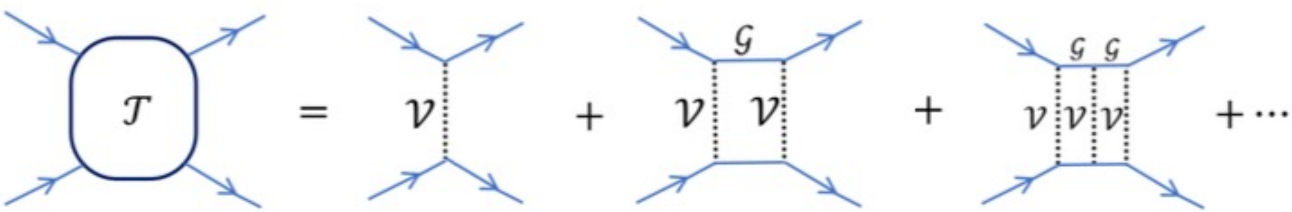
$$\mathcal{L}^{(\mathrm{s})} = +g_{\mathrm{s}}\bar{\psi}\psi\cdot\phi^{(\mathrm{s})}$$

$$\mathcal{L}^{(\mathrm{v})} = -g_{\mathrm{v}}\bar{\psi}\gamma^\mu\psi\cdot\phi_\mu^{(\mathrm{v})} - \frac{f_{\mathrm{v}}}{2M_{\mathrm{p}}}\bar{\psi}\sigma^{\mu\nu}\psi\cdot\partial_\mu\phi_\nu^{(\mathrm{v})}$$



$$V(\boldsymbol{q}',\boldsymbol{q}) = -\sum_{\text{all mesons}} \mathcal{F}_\alpha^2\,\bar{u}_1(\boldsymbol{q}')\Gamma_\alpha^{(1)}u_1(\boldsymbol{q})\frac{P_\alpha}{(\boldsymbol{q}'-\boldsymbol{q})^2+m_\alpha^2}\bar{u}_2(-\boldsymbol{q}')\Gamma_\alpha^{(2)}u_2(-\boldsymbol{q})$$

Bethe-Brueckner-Goldstone expansion



$$G(\boldsymbol{q}',\boldsymbol{q}|\boldsymbol{P},W) = V(\boldsymbol{q}',\boldsymbol{q}) + \int \frac{d^3k}{(2\pi)^3}V(\boldsymbol{q}',\boldsymbol{k})\,\frac{M^*_{\boldsymbol{P}+\boldsymbol{k}}M^*_{\boldsymbol{P}-\boldsymbol{k}}}{E^*_{\boldsymbol{P}+\boldsymbol{k}}E^*_{\boldsymbol{P}-\boldsymbol{k}}}\,\frac{Q(\boldsymbol{k},\boldsymbol{P})}{W - E_{\boldsymbol{P}+\boldsymbol{k}} - E_{\boldsymbol{P}-\boldsymbol{k}}}\,G(\boldsymbol{k},\boldsymbol{q}|\boldsymbol{P},W)$$

Theoretical framework

Self-energy of in-medium nucleon with positive energy states (PESs)

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta(M+\mathcal{U})]\psi(\boldsymbol{p},\lambda)=E_{\boldsymbol{p}}\psi(\boldsymbol{p},\lambda) \qquad \mathcal{U}(|\boldsymbol{p}|)=U_S(p)+\gamma^0 U_0(p)+\boldsymbol{\gamma}\cdot\hat{p}U_V(p)$$

$$\psi(\boldsymbol{p},\lambda)=u(\boldsymbol{p},\lambda)=\sqrt{\frac{E_p^*+M^*}{2M^*}}(1,\boldsymbol{\sigma}\cdot\boldsymbol{p}/2M)^T|\lambda\rangle \qquad \bar{u}u=1$$

$$M^*=M+U_S \qquad E_p^*=\sqrt{(U_V+\boldsymbol{p})^2+M^{*2}} \qquad E_p=E_p^*+U_0$$

Momentum-independence approximation (mom. ind. app.)

$$\mathcal{U}=U_S+\gamma^0 U_V \cancel{+\boldsymbol{\gamma}\cdot\hat{p}U_V} \qquad \Sigma(p)=\frac{M^*}{E_p^*}U_S+U_0$$

$$\Sigma(p)=\sum_{\tau,\lambda'}\int^{k_F}\frac{d\boldsymbol{p}'}{(2\pi)^3}\langle\bar{u}(\boldsymbol{p},1/2)\bar{u}(\boldsymbol{p}',\lambda')|G|\,\bar{u}(\boldsymbol{p},1/2)\bar{u}(\boldsymbol{p}',\lambda')\rangle$$

Projection method

$$G\ (\text{or}\ \ V)=\sum_{I=1,\cdots 5}\Gamma_I F_I=F_S\mathrm{S}+F_V\mathrm{V}+F_T\mathrm{T}+F_A\mathrm{A}+F_P\mathrm{P}$$

S, V, T, A, P:

$$\Gamma_I\in\{1\otimes 1,\gamma^\mu\otimes\gamma_\mu,\sigma^{\mu\nu}\otimes\sigma_{\mu\nu},\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu,\gamma^5\otimes\gamma^5\}$$

$$U_S(p)=\sum_{\tau'}\int^{k_F}\frac{d\boldsymbol{p}'}{(2\pi)^3}\frac{M^*}{E_{p'}^*}F_S \qquad p^{*\mu}U_\mu(p)=-\sum_{\tau'}\int^{k_F}\frac{d\boldsymbol{p}'}{(2\pi)^3}\frac{p_\mu'^*\,p^{*\mu}}{E_{p'}^*}F_V$$

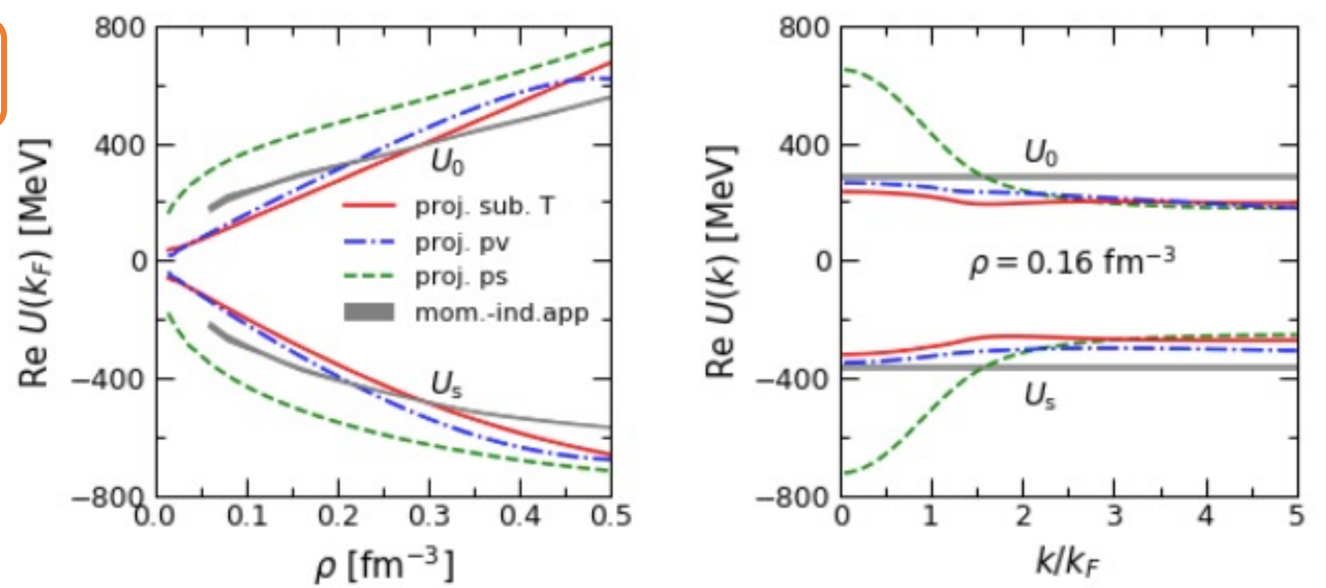
Self-energy of in-medium nucleon with PESs

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta(M + \mathcal{U})]\psi(\boldsymbol{p},\lambda) = E_{\boldsymbol{p}}\psi(\boldsymbol{p},\lambda) \qquad \mathcal{U}(|\boldsymbol{p}|) = U_S(p) + \gamma^0 U_0(p) + \boldsymbol{\gamma}\cdot\hat{p}U_V(p)$$

$$\psi(\boldsymbol{p},\lambda) = u(\boldsymbol{p},\lambda) = \sqrt{\frac{E_p^* + M^*}{2M^*}}(1, \boldsymbol{\sigma}\cdot\boldsymbol{p}/2M)^T|\lambda\rangle \qquad \bar{u}u = 1$$

$$M^* = M + U_S \qquad E_p^* = \sqrt{(U_V + \boldsymbol{p})^2 + M^{*2}} \qquad E_p = E_p^* + U_0$$

Ambiguities



T Gross-Boelting, et al., NuclPhysA648, 105 (1999)

$$U_S(p) = \sum_{\tau'}\int \frac{\cdot}{(2\pi)^3}\frac{\cdot}{E^*_{p'}}F_S \qquad p^{\mu}U_{\mu}(p) = -\sum_{\tau'}\int \frac{\cdot}{(2\pi)^3}\frac{\cdot}{E^*_{p'}}F_V$$

Theoretical framework

Full Dirac space with both PESs & negative energy states (NESs)

$$\psi(\boldsymbol{p},\lambda) = \begin{cases} u(\boldsymbol{p},\lambda) \\ v(\boldsymbol{p},\lambda) = \gamma^5 u(\boldsymbol{p},\lambda) \end{cases} \qquad (\bar{v}v = -1, \qquad \bar{v}u = \bar{u}v = 0)$$

$$M^* = M + U_S \qquad E_p^* = \sqrt{(\boldsymbol{p} + U_V)^2 + M^{*2}}$$

$$E_p^- = -E_p^* + U_0 \qquad E_p^+ = E_p^* + U_0$$

$$G = \begin{bmatrix} \langle \bar{u}\bar{u}|G|uu\rangle & \langle \bar{v}\bar{u}|G|uu\rangle \\ \langle \bar{u}\bar{u}|G|vu\rangle & \langle \bar{v}\bar{u}|G|vu\rangle \end{bmatrix} = \begin{bmatrix} G^{++++} & G^{-+++} \\ G^{++-+} & G^{-+-+} \end{bmatrix}$$

$$\Sigma^{hh'} = \sum_{\tau,\lambda'} \int^{k_F} \frac{d\boldsymbol{p}'}{(2\pi)^3} \langle \bar{u}(\boldsymbol{p},1/2)\bar{u}(\boldsymbol{p}',\lambda')|G^{h+h'+}|u(\boldsymbol{p},1/2)u(\boldsymbol{p}',\lambda')\rangle \qquad (h,h' = \pm)$$

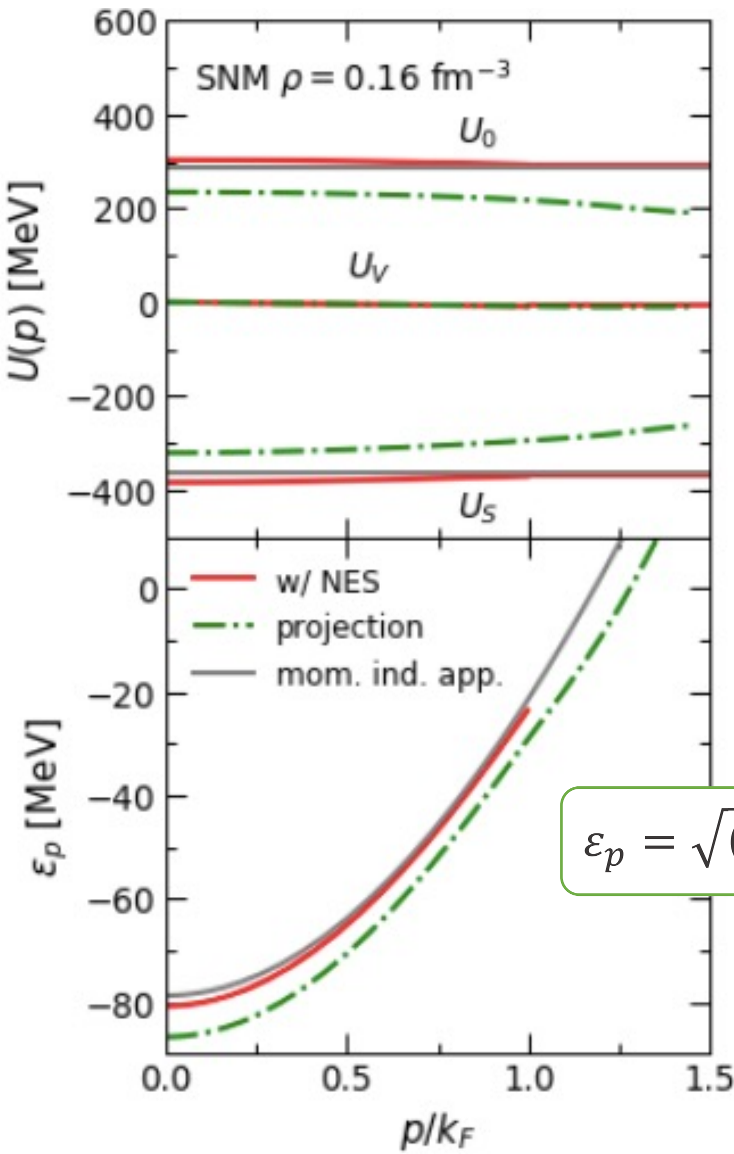
$$U_S(p) = \frac{\Sigma^{++}(p) - \Sigma^{--}(p)}{2}$$

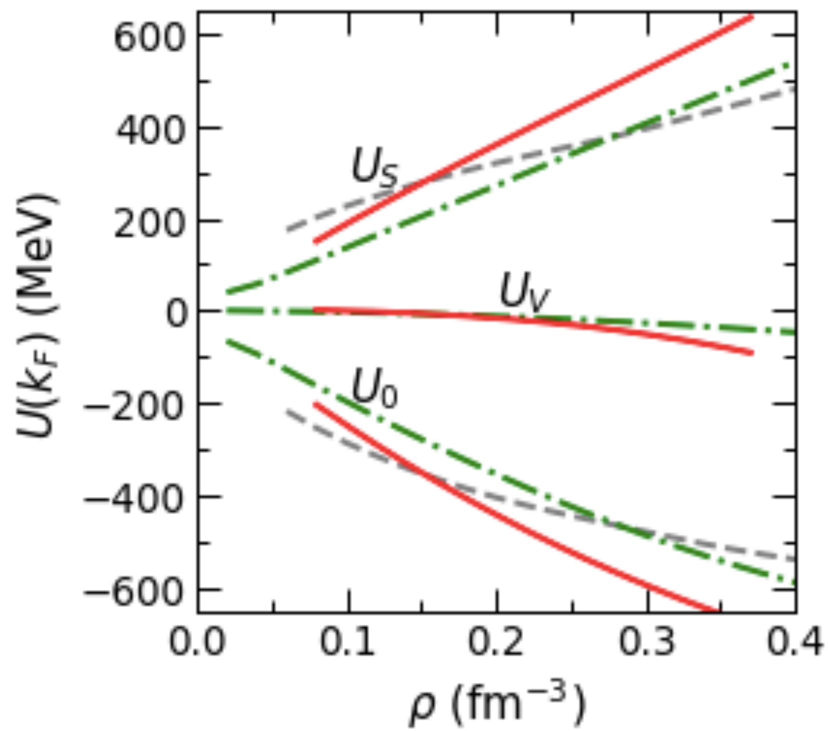
$$U_0(p) = \frac{E_p^*}{M_p^*}\frac{\Sigma^{++}(p) + \Sigma^{--}(p)}{2} - \frac{p^*}{M_p^*}\Sigma^{-+}(p)$$

$$U_V(p) = -\frac{p^*}{M_p^*}\frac{\Sigma^{++}(p) + \Sigma^{--}(p)}{2} + \frac{E_p^*}{M_p^*}\Sigma^{-+}(p)$$

S B Wang, et al., PhysRevC103, 054319 (2021)

Single-nucleon potential





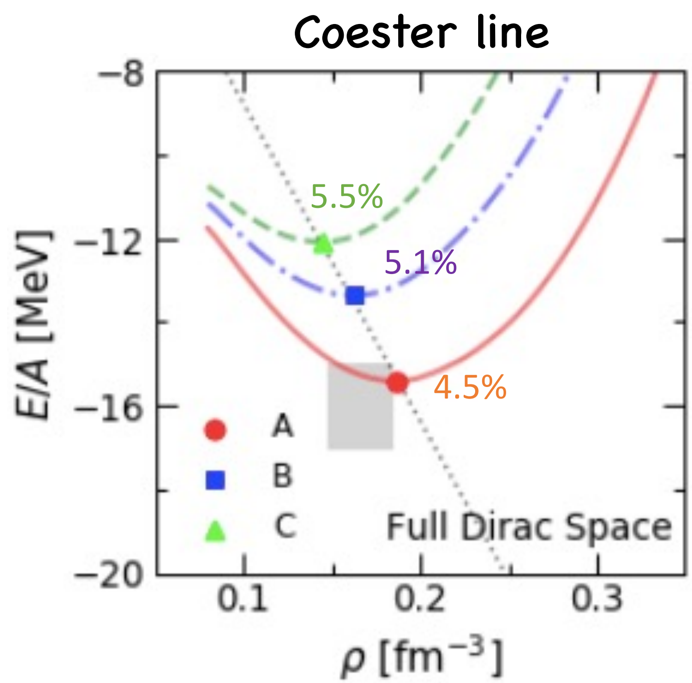
$$\varepsilon_p = \sqrt{(p + U_V)^2 + (M + U_S)^2} + U_0 - M$$

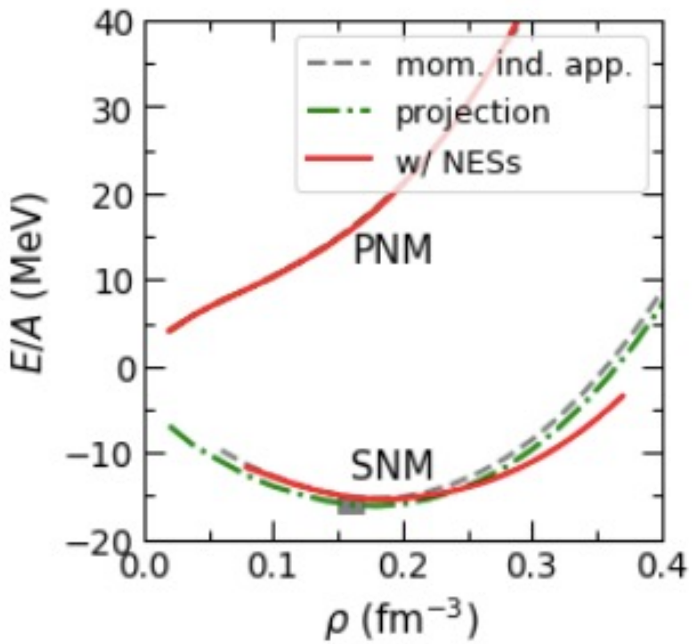
Saturation properties

$$E/A = \frac{1}{\rho}\sum_{s}\int_0^{k_F}\frac{d^3p}{(2\pi)^3}\frac{M_{\boldsymbol{p}}^*}{E_{\boldsymbol{p}}^*}\langle\bar{u}(\boldsymbol{p},s)|\boldsymbol{\gamma p}+M|u(\boldsymbol{p},s)\rangle - M$$

$$+\frac{1}{2\rho}\sum_{s,s'}\int_0^{k_F}\frac{d^3p}{(2\pi)^3}\int_0^{k_F}\frac{d^3p'}{(2\pi)^3}\frac{M_{\boldsymbol{p}}^*}{E_{\boldsymbol{p}}^*}\frac{M_{\boldsymbol{p}'}^*}{E_{\boldsymbol{p}'}^*}$$

$$\times\ \langle\bar{u}(\boldsymbol{p},s)\bar{u}(\boldsymbol{p}',s')|\bar{G}^{++++}(W)|u(\boldsymbol{p},s)u(\boldsymbol{p}',s')\rangle$$





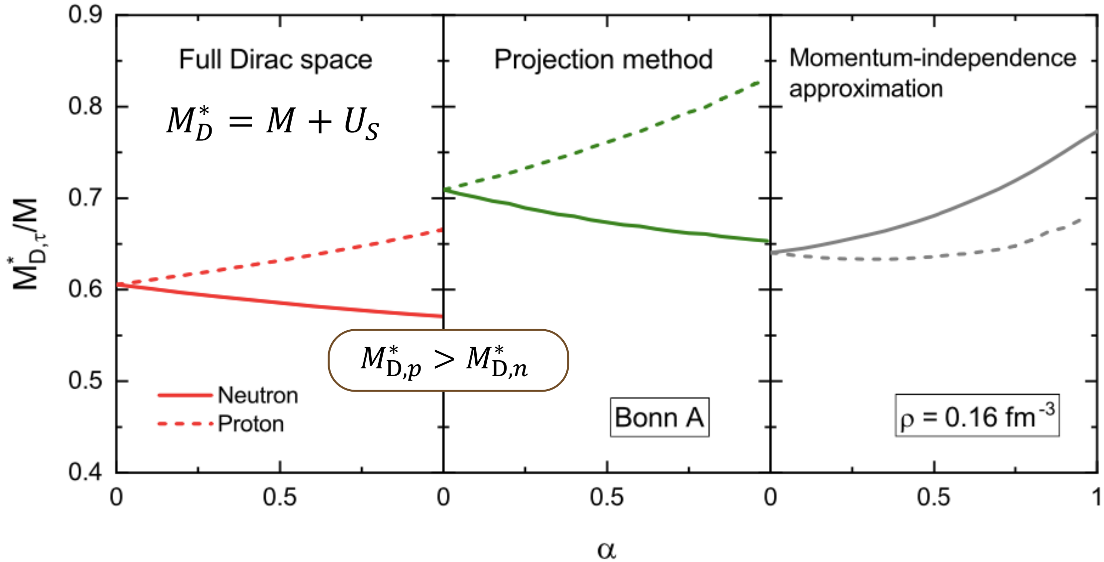
	ρ_0 [fm^{-3}]	E/A [MeV]	M_D^*/M	K_∞ [MeV]
Mom.Ind.App.	0.178	-15.38	0.59	273
Projection	0.179	-16.15	0.68	254
Full Dirac Space	0.186	-15.40	0.55	258
Emp.	0.16±0.01	-16±1		240± 20

S B Wang, et al., PhysRevC103, 054319 (2021)

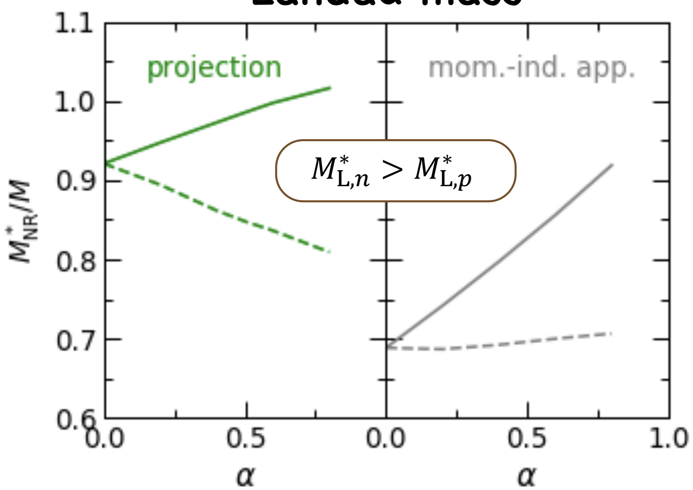
S B Wang, et al., arXiv: 2203.05397

The effective mass

Dirac mass



Landau mass



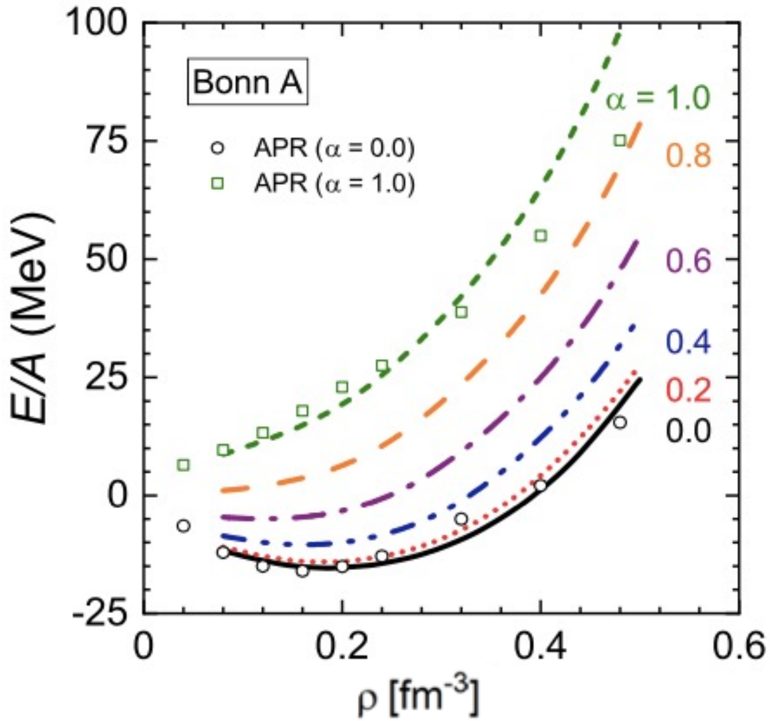
$$\frac{M_L^*}{M} = \left[1 + \frac{M}{p}\frac{dU_{se}}{dp}\right]^{-1}_{p=k_F}$$

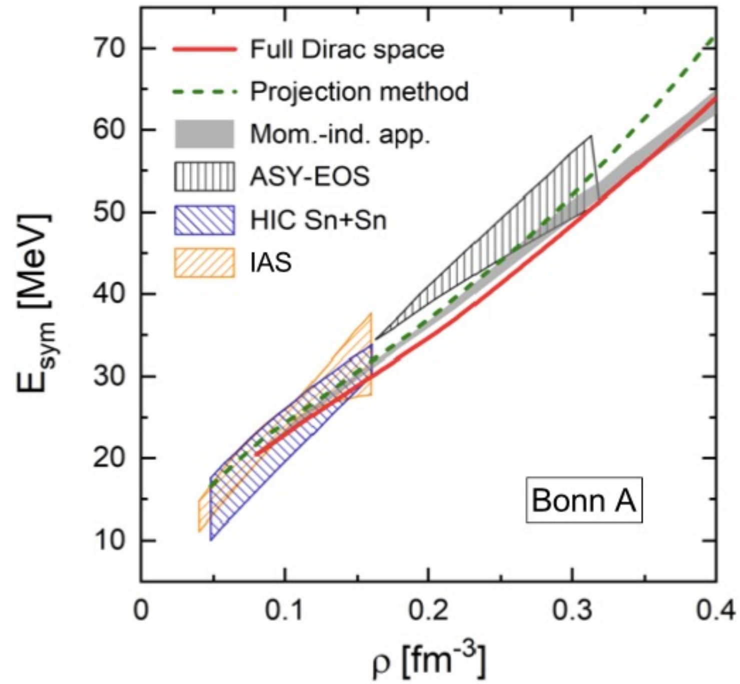
$$\left(\frac{p^2}{2M} + U_{se}(p)\right)\varphi(\boldsymbol{p}, s) = \left(e + \frac{e^2}{2M}\right)\varphi(\boldsymbol{p}, s)$$

$$U_{se} = U_S + \frac{E_p}{M}U_0 + \frac{pU_V}{M} + \frac{U_S^2 - U_0^2 + U_V^2}{2M}$$

H Tong, et al., AstroPhysJ930, 137 (2022)

Symmetry energy





$$\frac{E}{A}(\rho,\alpha) = \frac{E}{A}(\rho,0) + \alpha^2 E_{\mathrm{sym}} + \cdots$$

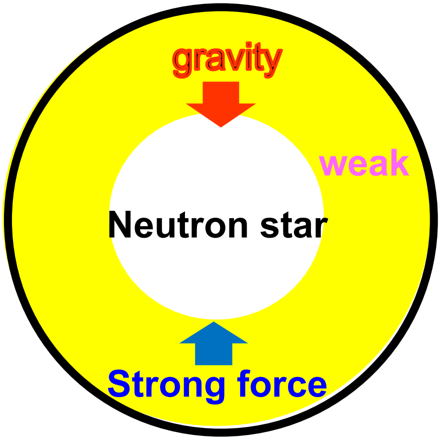
$$E_{\mathrm{sym}} = \left(\frac{1}{2}\frac{\partial^2 E/A}{\partial \alpha^2}\right)_{\alpha=0}$$

$$= E_{\mathrm{sym}}\,(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\mathrm{sym}}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2$$

	E_{sym} [MeV]	L[MeV]	K_{sym} [MeV]
Mom.Ind.App.	33.2	67.3	-65.7
Projection	34.7	68.8	-70.1
Full Dirac Space	0.188	33.1	65.2
Emp.	32±2	88±25	

Neutron star properties

Neutron star core matter in beta-equilibrium

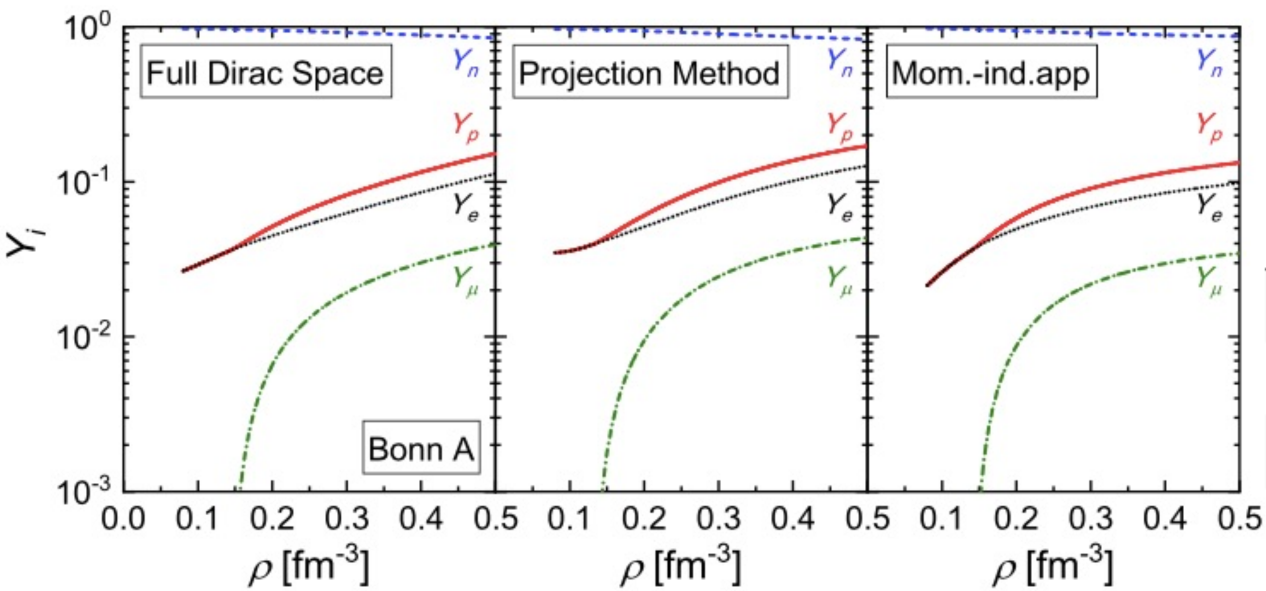


$$\mu_p = \mu_n - \mu_e \qquad \mu_\mu = \mu_e \qquad \mu_i = \frac{\partial \varepsilon / \rho}{\partial Y_i}$$

$$\rho_p = \rho_e + \rho_\mu$$

$$\varepsilon = \rho[E(\rho, 1 - 2Y_p)/A + Y_p M_p + (1 - Y_p)M_n] + \varepsilon_e + \varepsilon_\mu$$

$$P = -\frac{\partial(\varepsilon/\rho)}{\partial(1/\rho)} = \rho\frac{\partial \varepsilon}{\partial \rho} - \varepsilon$$



	ρ_μ [fm^{-3}]	ρ_{DU} [fm^{-3}]	$Y_{p, DU}$
MIA	0.142	0.464	0.127
Prj	0.132	0.372	0.126
FDS	0.144	0.432	0.127

H Tong et al., AstroPhysJ930, 137 (2022)

Neutron star properties

H Tong et al., AstroPhysJ930, 137 (2022)

Tolman-Oppenheimer-Volkov equations

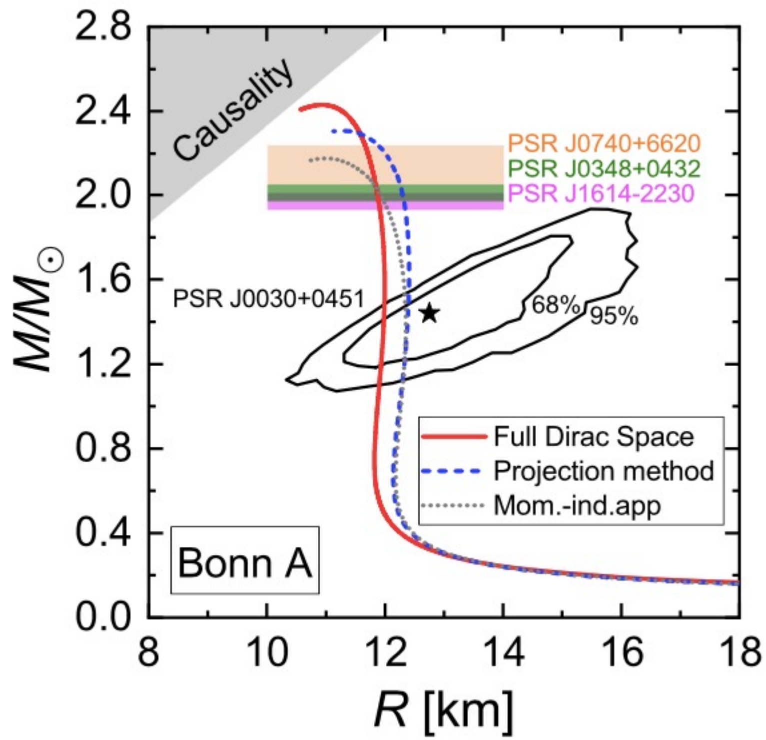
$$\frac{dP(r)}{dr} = -\frac{[P(r)+\varepsilon(r)][M(r)+4\pi r^3 P(r)]}{r[r-2M(r)]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r) \qquad (G = c = 1)$$

$$r\frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2 Q(r) = 0$$

$$F(r) = \left[1 - \frac{2M(r)}{r}\right]^{-1}\{1 - 4\pi r^2[\varepsilon(r) - P(r)]\}$$

$$Q(r) = \left\{4\pi\left[5\varepsilon(r) + 9P(r) + \frac{\varepsilon(r)+P(r)}{\frac{\partial P}{\partial \varepsilon}(r)}\right] - \frac{6}{r^2}\right\} \times\left[1 - \frac{2M(r)}{r}\right]^{-1} - \left[\frac{2M(r)}{r^2} + 2\times 4\pi r P(r)\right]^2 \times\left[1 - \frac{2M(r)}{r}\right]^{-2}.$$



	R_{max}/[km]	$M_{max}/M_{\odot}$
Mom.Ind.App.	10.98	2.18
Projection	11.27	2.31
Full Dirac space	10.93	2.43

Neutron star properties

S B Wang, et al., arXiv:2206.08579

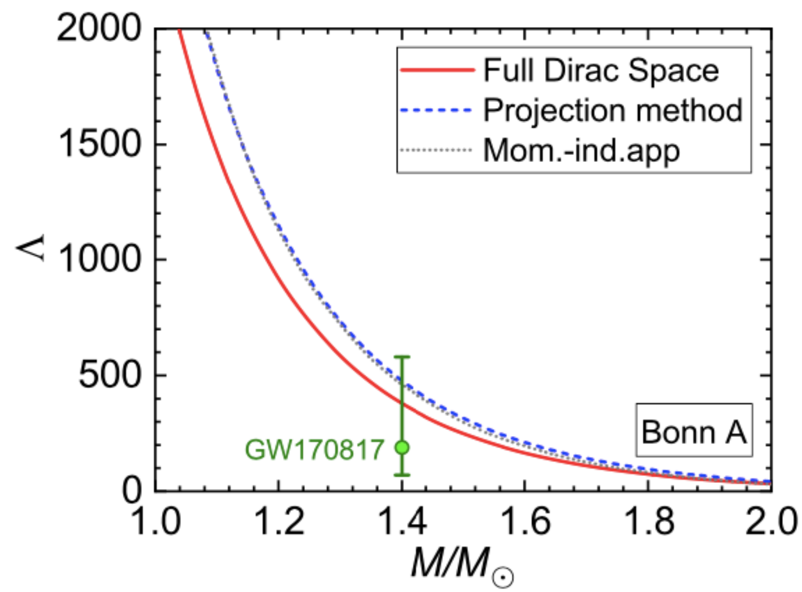
Tidal deformability

$$\Lambda = \frac{2}{3}k_2 C^{-5}$$

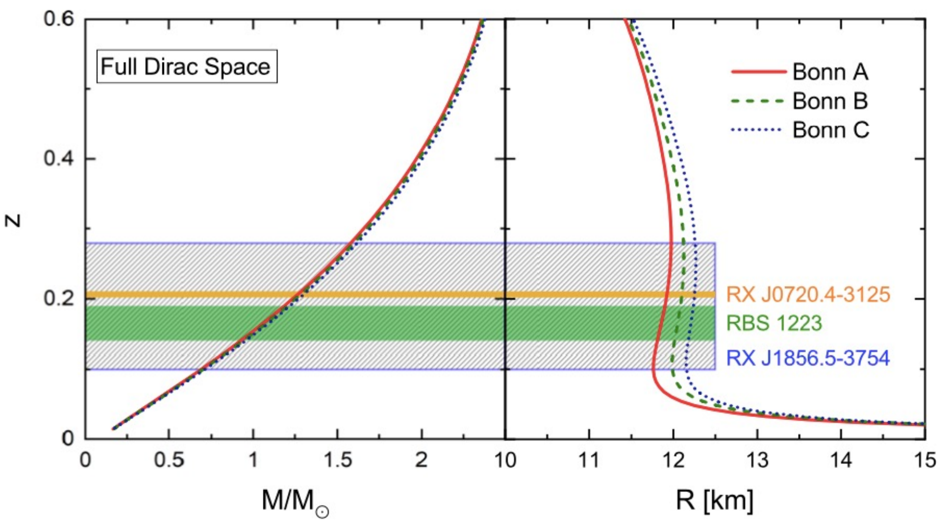
Love number

$$k_2 = \frac{8C^5}{5}(1-2C)^2[2-y_R+2C(y_R-1)] \times \{6C[2-y_R+C(5y_R-8)] + 4C^3[13-11y_R+C(3y_R-2)+2C^2(1+y_R)] + 3(1-2C)^2[2-y_R+2C(y_R-1)]\ln(1-2C)\}^{-1}$$

Compactness $C = M/R$



Surface redshift

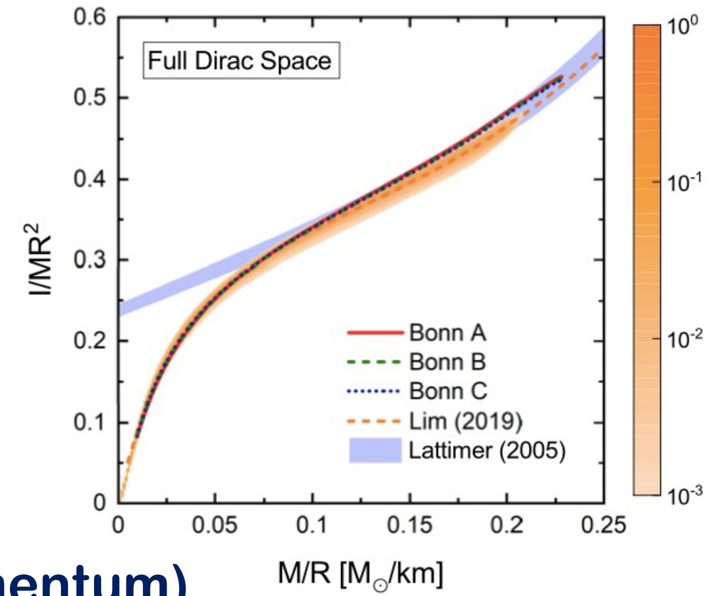


$$z = (1-C)^{-\frac{1}{2}} - 1$$

	z	M	R/[km]
RBS1223	$0.16^{+0.03}_{-0.02}$	$1.03^{+0.05}_{-0.11}$	$11.85^{+0.05}_{-0.04}$
RXJ1856.5-3754	$0.22^{+0.06}_{-0.12}$	$1.33^{+0.25}_{-0.64}$	$11.94^{+0.03}_{-0.18}$
RXJ0720.4-3125	$0.205^{+0.006}_{-0.003}$	$1.258^{+0.028}_{-0.014}$	$11.922^{+0.008}_{-0.004}$

Neutron star properties

S B Wang et al, arXiv:2206.08579

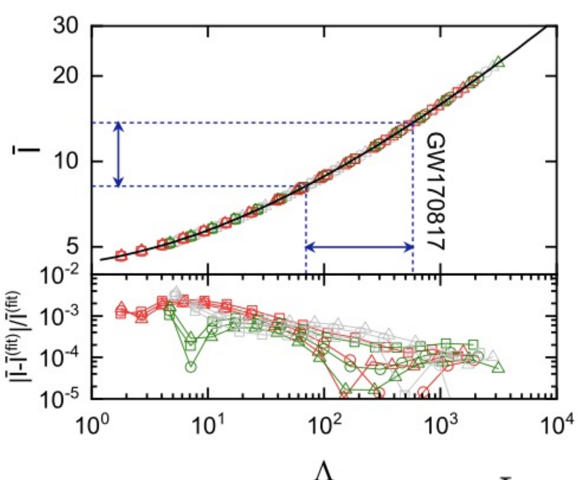


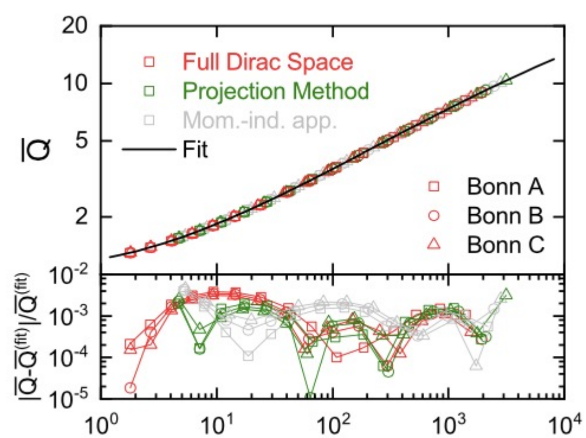
- **The momentum of inertia of slow-rotation neutron star**

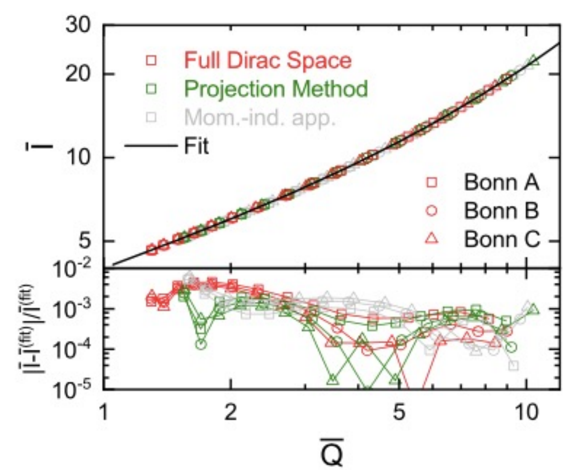
$$I = \frac{8\pi}{3}\int_0^R dr \cdot r^4 e^{-\nu(r)}\frac{\bar{\omega}(r)}{\Omega}\frac{\varepsilon + P}{\sqrt{1-\frac{2M}{r}}}$$

$$\nu(r) = \frac{1}{2}\ln\left(1-\frac{2M}{R}\right) - \int_r^R \frac{M(x)+4\pi x^3 P(x)}{x^2[1-2M(x)/x]}dx$$

- **I-Love-Q relations (Q as quadrupole momentum)**







$$\bar{I} \equiv \frac{I}{M^3}$$

$$\bar{Q} \equiv -\frac{QM}{(I\Omega)^2}$$

Summary & perspectives

Summary & perspectives

- **Relativistic Brueckner-Hartree-Fock (RBHF) theory in full Dirac space is constructed**

  In the full Dirac space, the single-nucleon potentials can be solely determined

  In full Dirac space, asymmetric matter predictions agree with those from projection method quantitively, such as Dirac mass splitting

- **RBHF in full Dirac space applied to neutron star**

  Full Dirac space predicts neutron star more compact for given neutron star mass

  Full Dirac space predictions also conform to neutron star universal relations

- ***Accomplish more with RBHF theory***

  Three hole-lines   Thermodynamically conserved RBHF theory

  Three-body force & relativistic effect

- ***Development of novel relativistic approaches***

Thank you for your attention