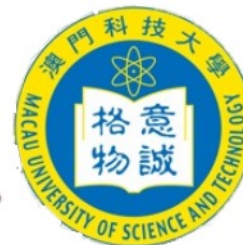


第一届“粤港澳”核物理论坛



Light mesons with one dynamical gluon on the light front

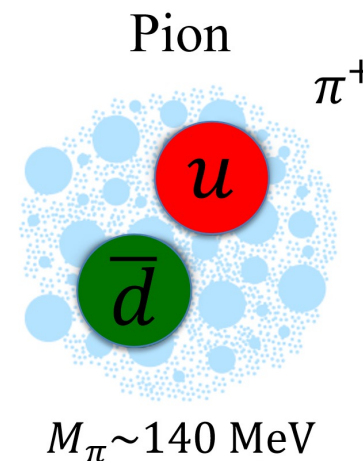
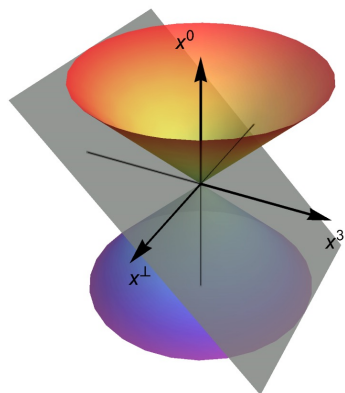
蓝江山

近代物理研究所

合作：付开宇、C. Mondal、赵行波、
J. P. Vary

基于：Physics Letters B 825 (2022) 136890

2022年7月4日 广东·珠海 online



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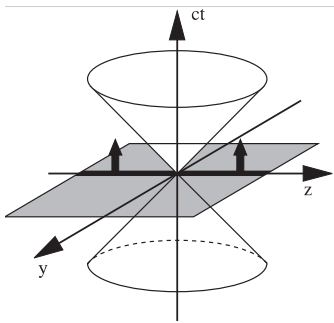
- 基矢光前量子化方法
- 简要回顾考虑有效NJL的轻介子
- 考虑一个动力学胶子的轻介子
- 总结展望

基矢光前量子化(BLFQ)

[Dirac, 1949]

等时量子化

$$t \equiv x^0$$



$$x^1, x^2, x^3$$

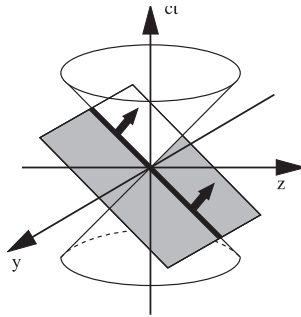
$$P^0, \vec{P}$$

$$i \frac{\partial}{\partial t} |\varphi(t)\rangle = H |\varphi(t)\rangle$$

$$P^0 = \sqrt{m^2 + \vec{P}^2}$$

光前量子化

$$t \equiv x^+ = x^0 + x^3$$



$$x^- = x^0 - x^3, \\ x^\perp = x^{1,2}$$

$$P^- = P^0 - P^3, \\ P^+ = P^0 + P^3, P^\perp = P^{1,2}$$

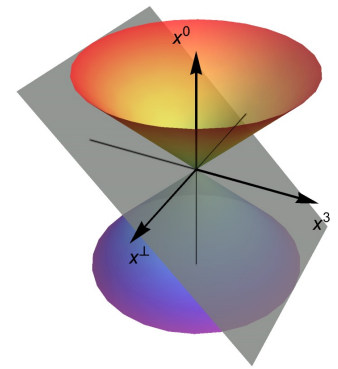
$$i \frac{\partial}{\partial x^+} |\varphi(x^+)\rangle = \frac{1}{2} P^- |\varphi(x^+)\rangle$$

$$P^- = \frac{m^2 + P_\perp^2}{P^+}$$

差异:

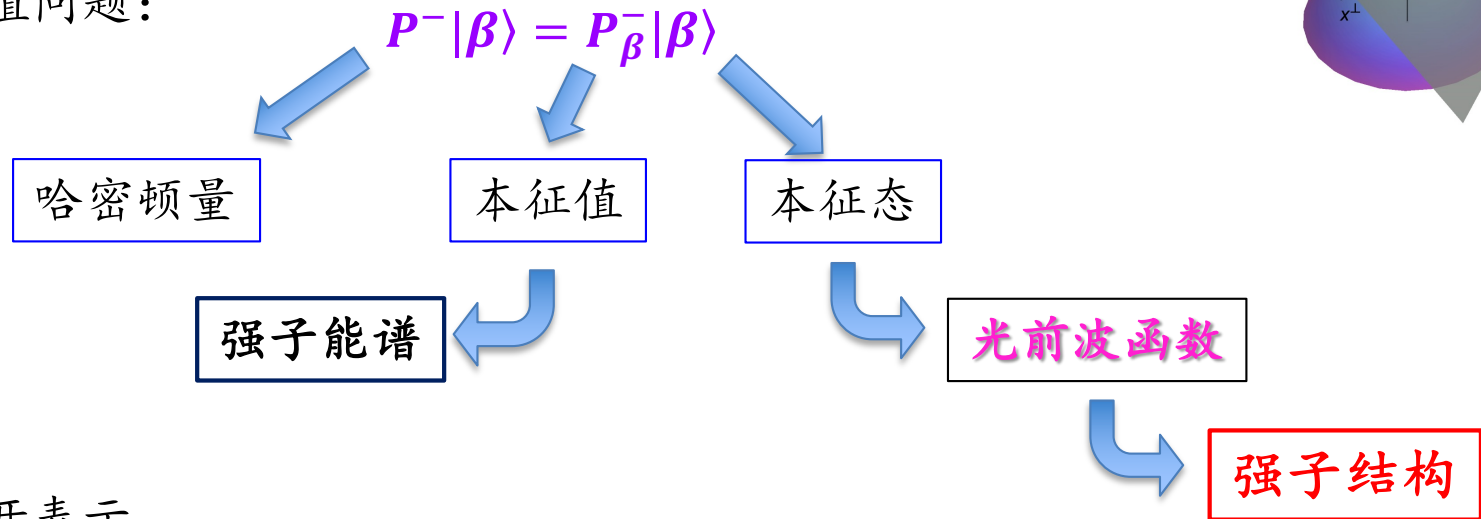
- 波函数不随参考系变化
- 在光锥规范下, 纵向规范链是平庸的
- 波函数可直接用来计算强子结构
- 哈密顿量 P^- 没有平方根, 可用Fock展开来研究强子结构

基矢光前量子化(BLFQ)



[Vary et al, 2008]

- 非微扰本征值问题:



- Fock空间展开表示

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + \dots$$

- 基矢的离散化表示

- 横向: 二维谐振子 $\Phi_{n,m}^b(\vec{p}_{\perp})$.
- 纵向: 平面波 e^{-ikx} .
- 基矢截断: $\sum_i (2n_i + |m_i| + 1) \leq N_{max},$
 $\sum_i k_i = K.$

N_{max}, K 对应横向和纵向截断参数~分别对应于紫外截断和红外截断.

近几年该方法得到了长足发展

Y. Liu et al, PRD 105(2022) 094018	Z. Hu et al, PRD 103(2021) 036005
S. Nair et al, PLB 827(2022) 137005	M. Li et al, PRD 101(2020) 076016
Z. Kuang et al, PRD 105(2022) 094028	J. Lan et al, PRD 102(2020) 014020
J. Lan et al, PLB 825 (2022) 136890	C. Mondal et al, PRD 102 (2020) 016008
L. Adhikari et al, PRD 104 (2021) 114019	J. Lan et al, PRD 101 (2020) 034024
C. Mondal et al, PRD 104(2021) 094034	J. Lan al et al, PRL 122 (2019) 172001
S. Xu et al, PRD 104(2021) 094036	

- 自旋: s , 总角动量在第三分量的投影守恒

$$\sum_i m_i + s_i = M_J$$

简要回顾

轻介子的部分子分布

$$|\text{meson}\rangle = |q\bar{q}\rangle + \dots$$

$$H_{\text{eff}} = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$

对角化哈密顿量



光前波函数

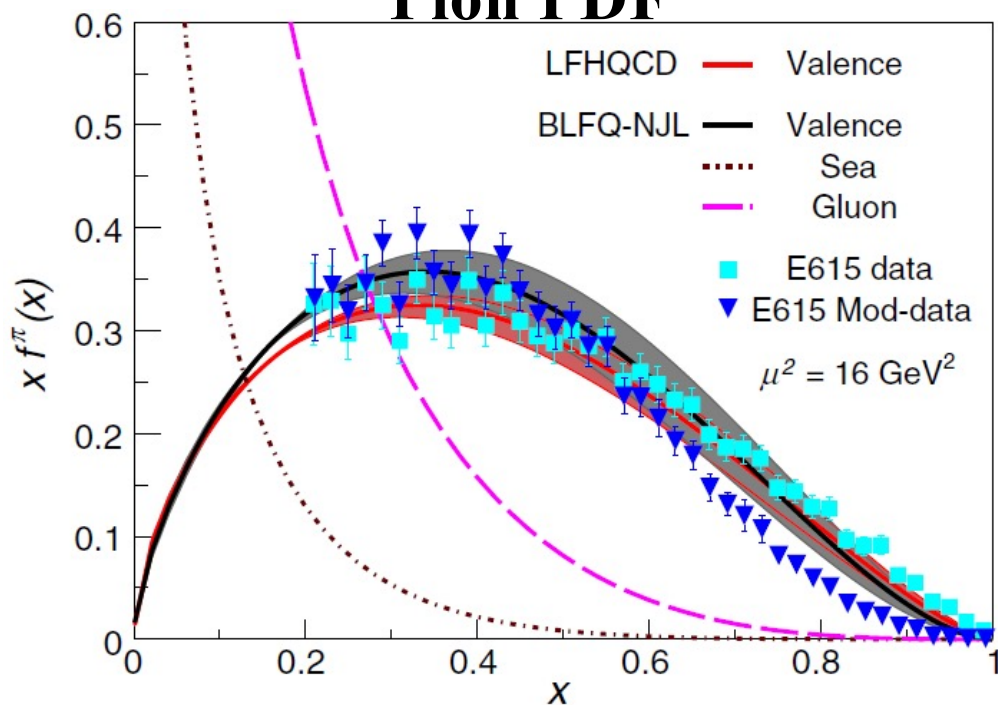


初始部分子分布

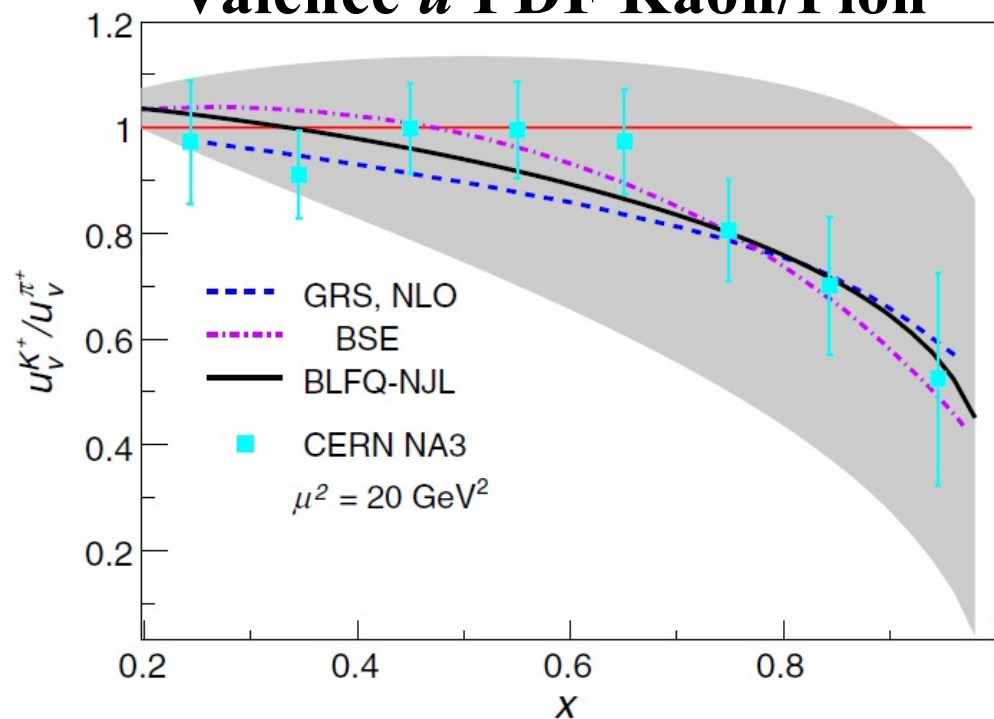


QCD演化

Pion PDF



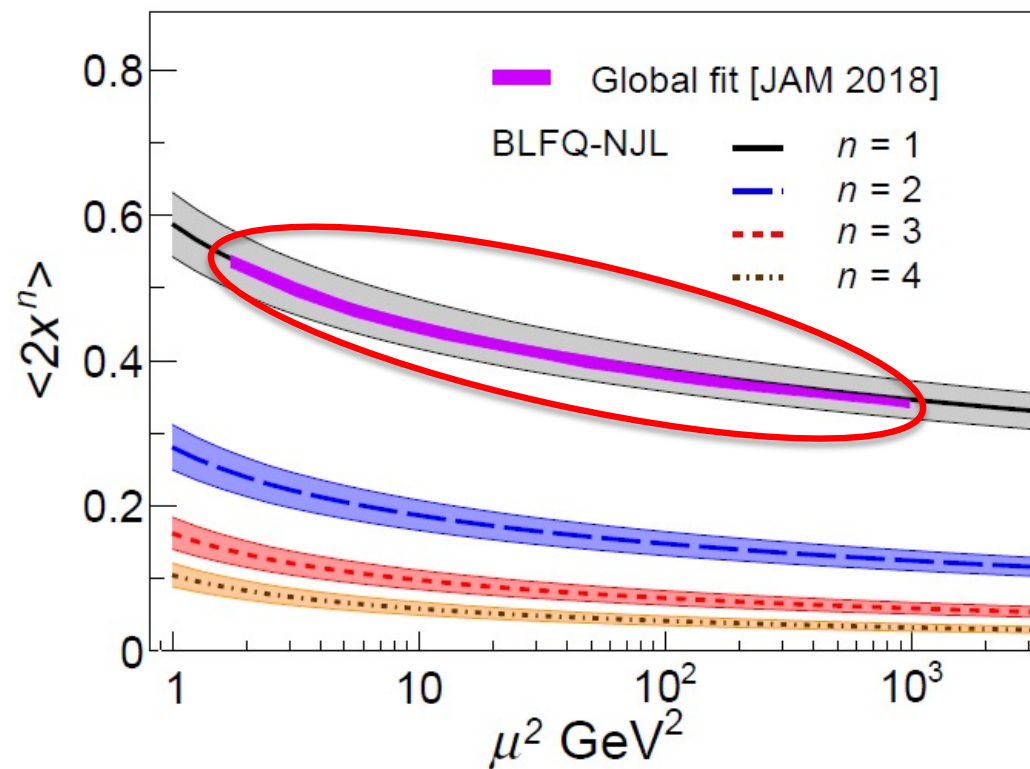
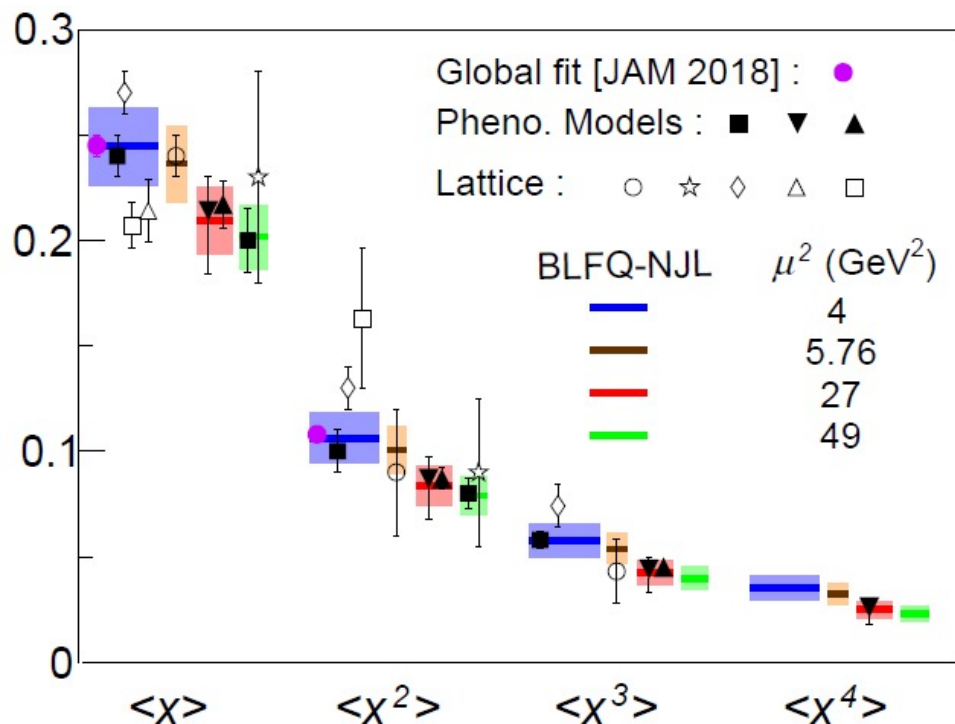
Valence u PDF Kaon/Pion



简要回顾

轻介子的结构

$$\langle x^n \rangle = \int_0^1 dx x^n f_{V,S,g}^\pi(x, \mu^2), \quad n = 1, 2, 3, 4.$$

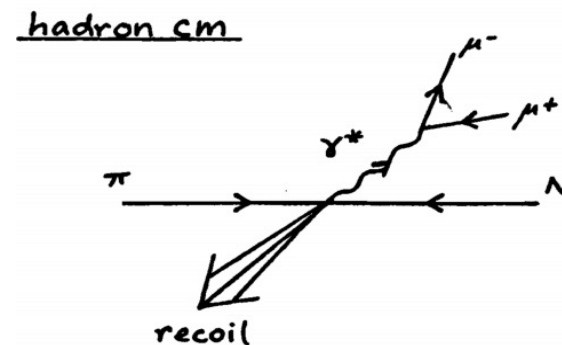


$\langle x \rangle @ 4 \text{ GeV}^2$	Valence	Gluon	Sea
BLFQ-NJL	0.489	0.398	0.113
[Ding <i>et. al.</i> , BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

与其他模型的结果相符

简要回顾

Drell-Yan微分截面



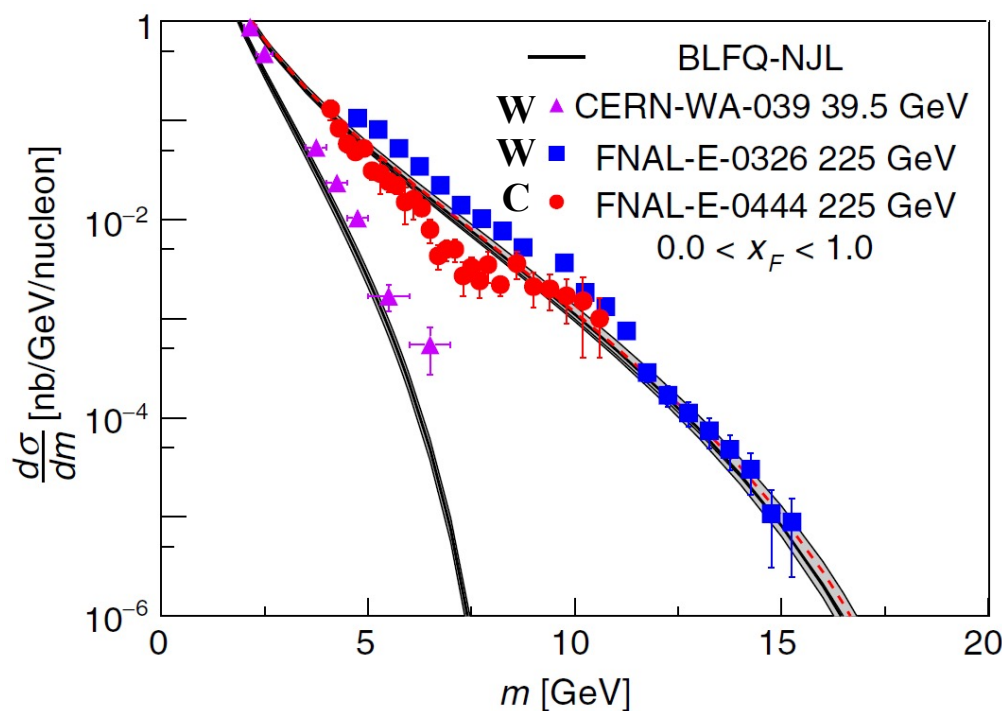
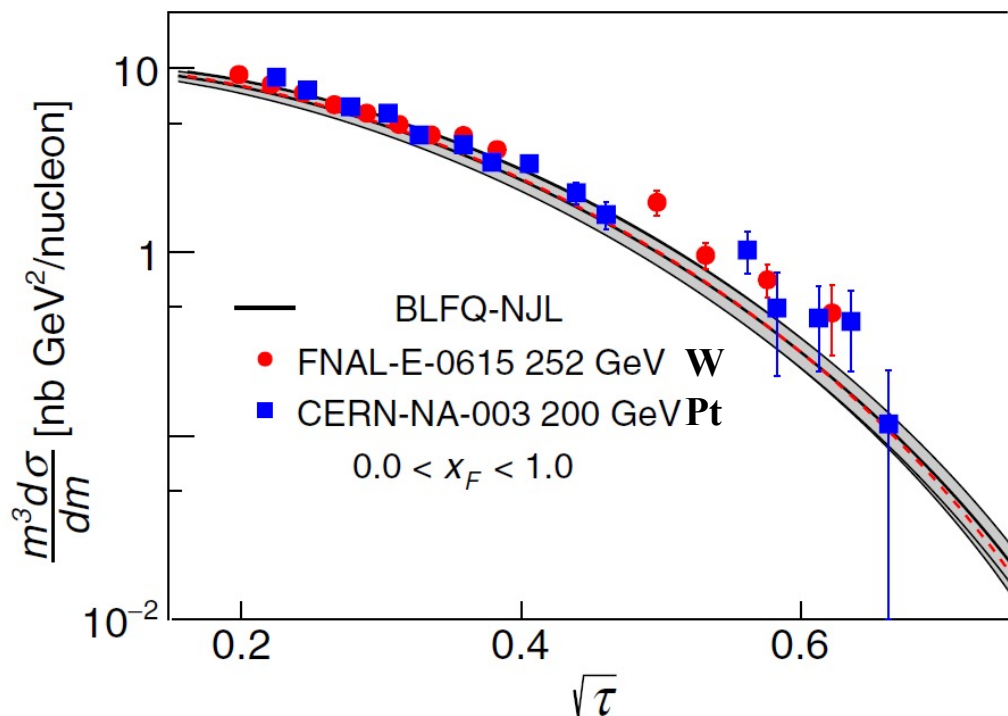
[S. D. Drell and T.-M. Yan, PRL (1970)]

[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]

[C. Anastasiou et al, PRL91(2003)182002]

$$\frac{m^3 d^2\sigma}{dm dY} = \frac{8\pi\alpha^2 m^2}{9 s} \sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$

[nCTEQ 2015]



与实验结果大致相符(FNAL E615, 326, 444, & CERN NA3, WA-039).

[Lan, et al, PRL 19']

$$|\text{meson}\rangle = |q\bar{q}\rangle + \dots$$

往前一步

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]



$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

H_{int}	$ q\bar{q}\rangle$	$ q\bar{q}g\rangle$
$\langle q\bar{q} $		
$\langle q\bar{q}g $		0

$$H_{\text{eff}} = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$



$$P^- = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x(x(1-x)\partial_x) + H_{\text{int}}$$

标准的光前哈密顿量

[Brodsky et al, 1998]

$$P_{-,LFQCD} = \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \psi - \frac{1}{2} \int d^3x A_a^i (i\partial^\perp)^2 A_a^i$$

$$+ g \int d^3x \bar{\psi} \gamma_\mu A^\mu \psi$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+} \gamma_\nu A^\nu \psi$$

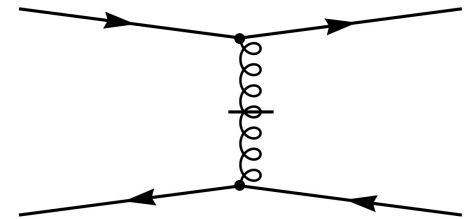
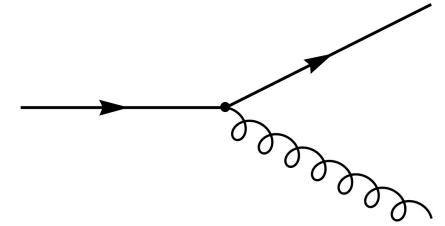
$$- i g^2 \int d^3x f^{abc} \bar{\psi} \gamma^+ T^c \psi \frac{1}{(i\partial^+)^2} (i\partial^+ A_a^\mu A_{\mu b})$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi$$

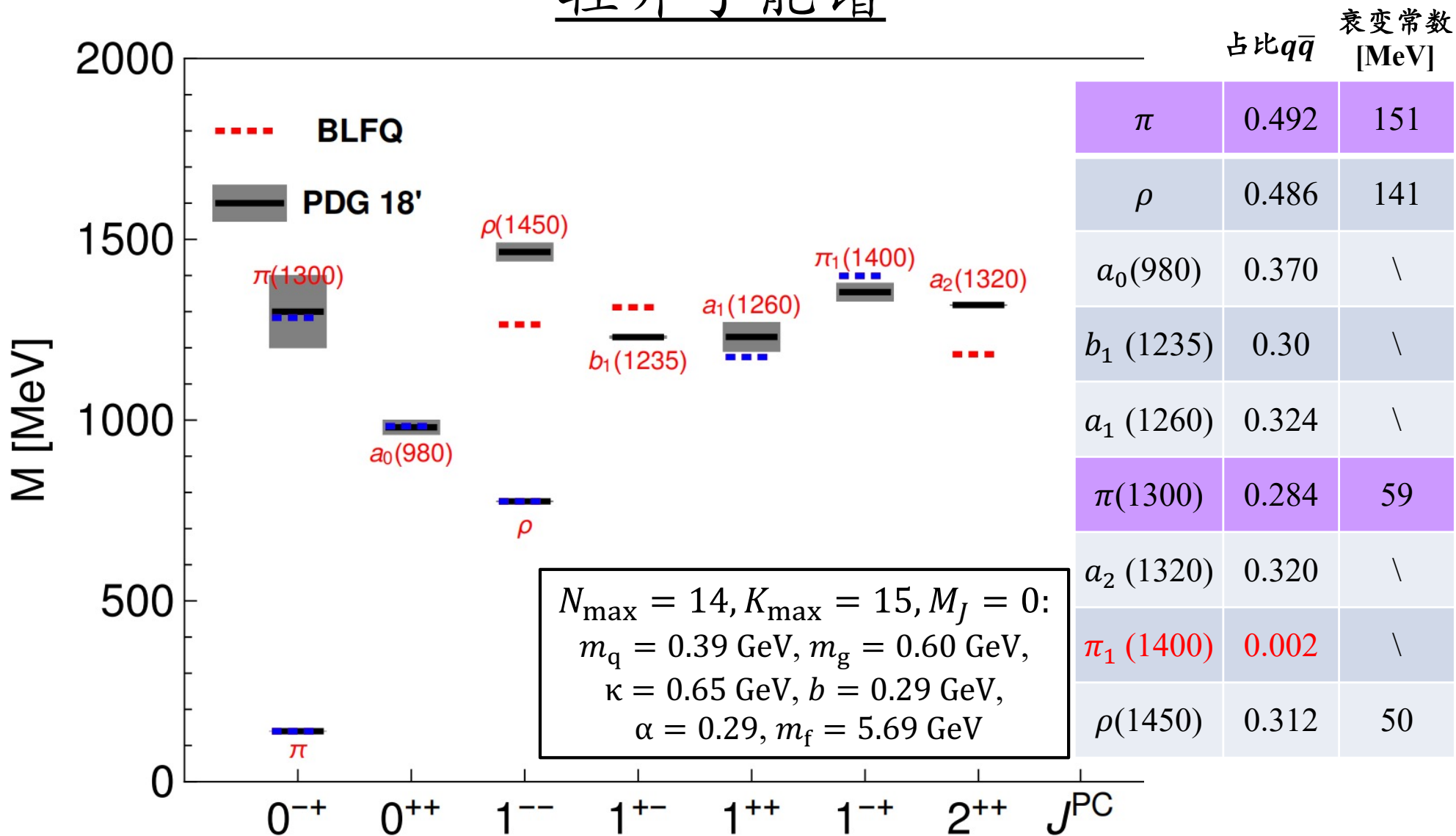
$$+ i g \int d^3x f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c$$

$$- \frac{1}{2} g^2 \int d^3x f^{abc} f^{ade} i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2} (i\partial^+ A_d^+ A_{ve})$$

$$+ \frac{1}{4} g^2 \int d^3x f^{abc} f^{ade} A_b^\mu A_c^\nu A_{\mu d} A_{ve}$$



轻介子能谱



$N_{\max} = 14, K_{\max} = 15, M_J = 0:$
 $m_q = 0.39 \text{ GeV}, m_g = 0.60 \text{ GeV},$
 $\kappa = 0.65 \text{ GeV}, b = 0.29 \text{ GeV},$
 $\alpha = 0.29, m_f = 5.69 \text{ GeV}$

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

通过蓝色的六个态确定模型参数

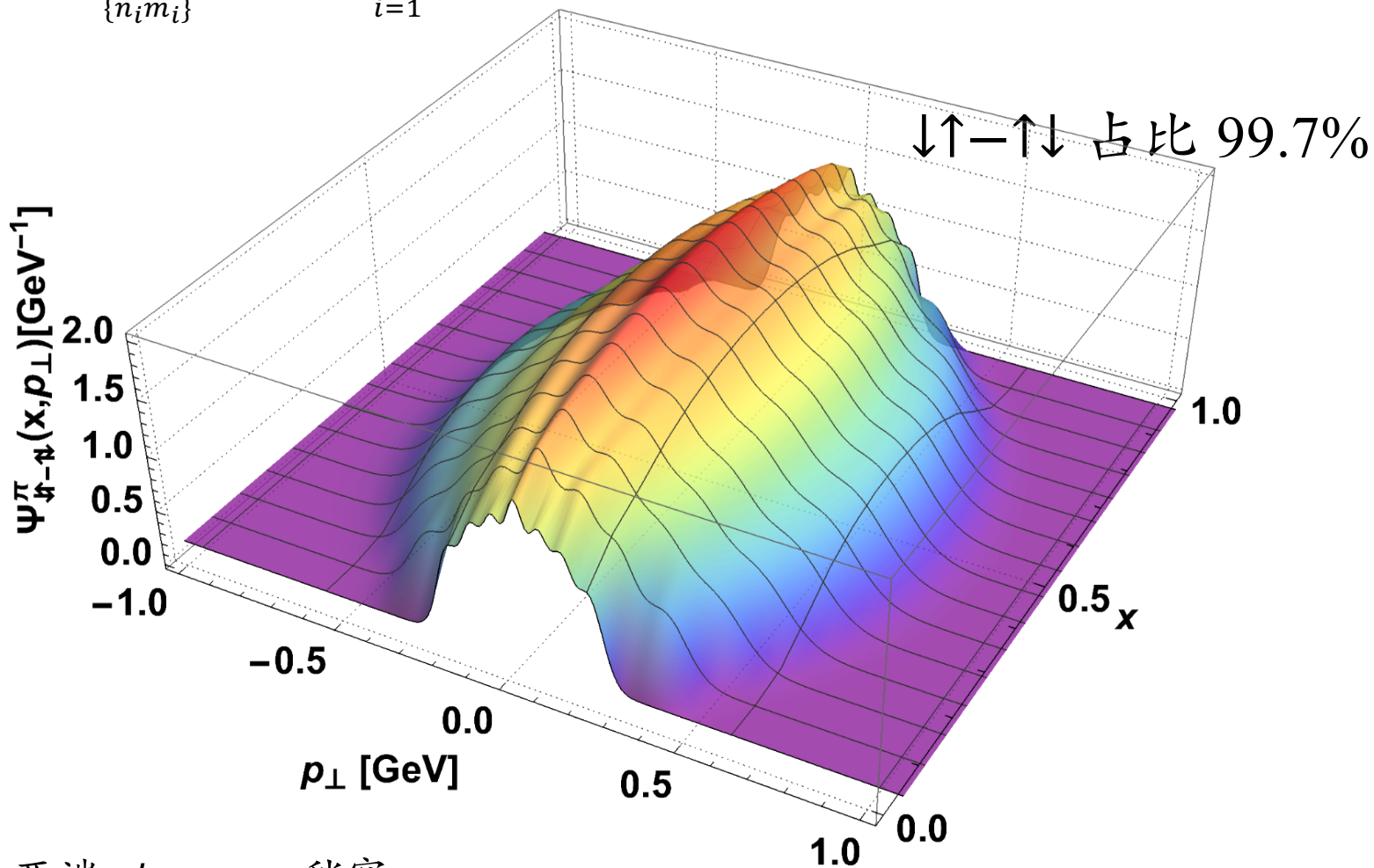
➤ $\pi_1(1400)$: $|q\bar{q}g\rangle$ 占主导

➤ $\pi(1300)$: 衰变常数小于pion的衰变常数

领头Fock空间中的波函数

$$\Psi_{\{x_i, \vec{p}_{\perp i}^2, \lambda_i\}}^{\mathcal{N}, M_J} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\bar{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



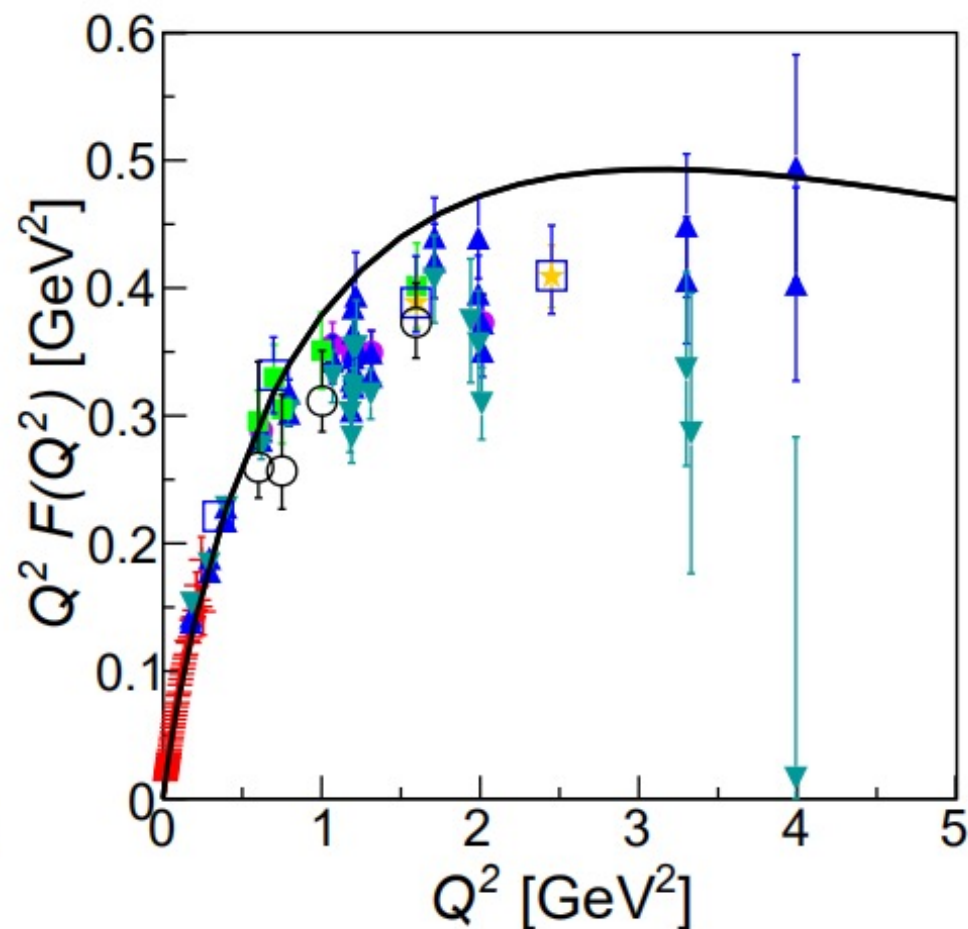
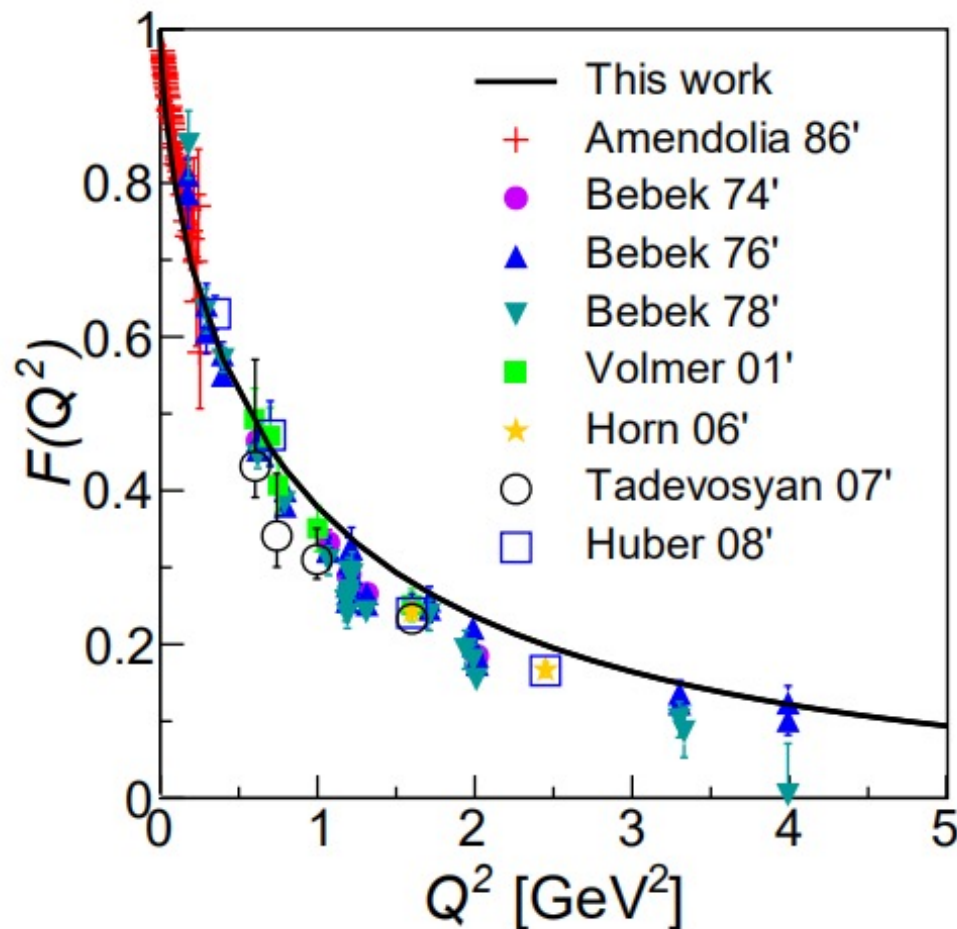
- 纵向的两端, $\psi \sim p_{\perp}$: 稍窄
- 纵向的中间, $\psi \sim p_{\perp}$: 稍宽

Pion介子的电磁形状因子

[Brodsky & de Teramond, PRD 77:056007 (2008)]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$\langle\Psi(p')|J_{EM}^+(0)|\Psi(p)\rangle = (p+p')^+ F(Q^2)$$



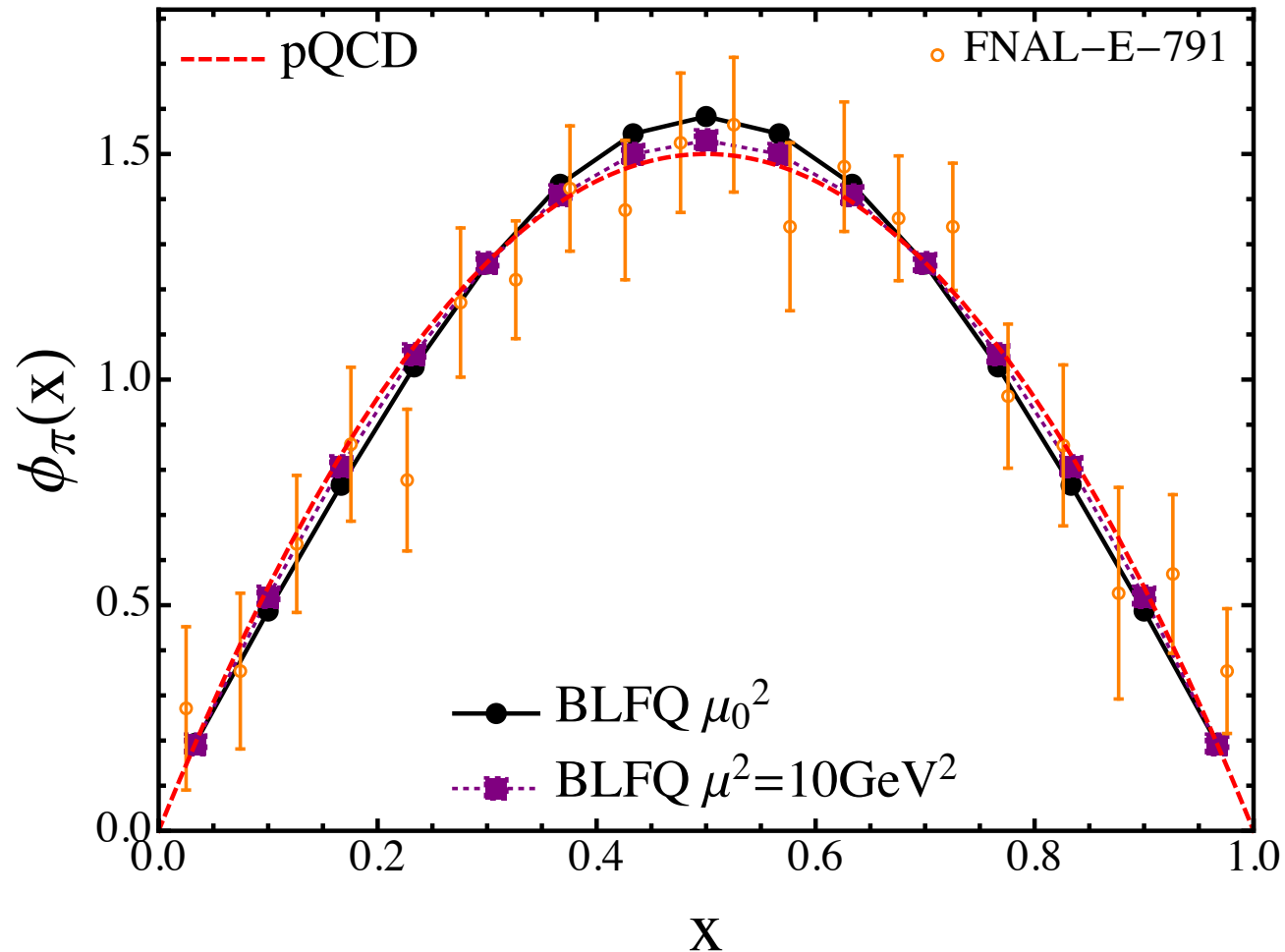
- 形状因子比较合理地与实验数据相符
- 在大 Q^2 区域, $F(Q^2) \propto 1/Q^2$

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

Pion介子的部分子分布振幅

[Ruiz Arriola & Broniowski, PRD 66:094016 (2002)]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

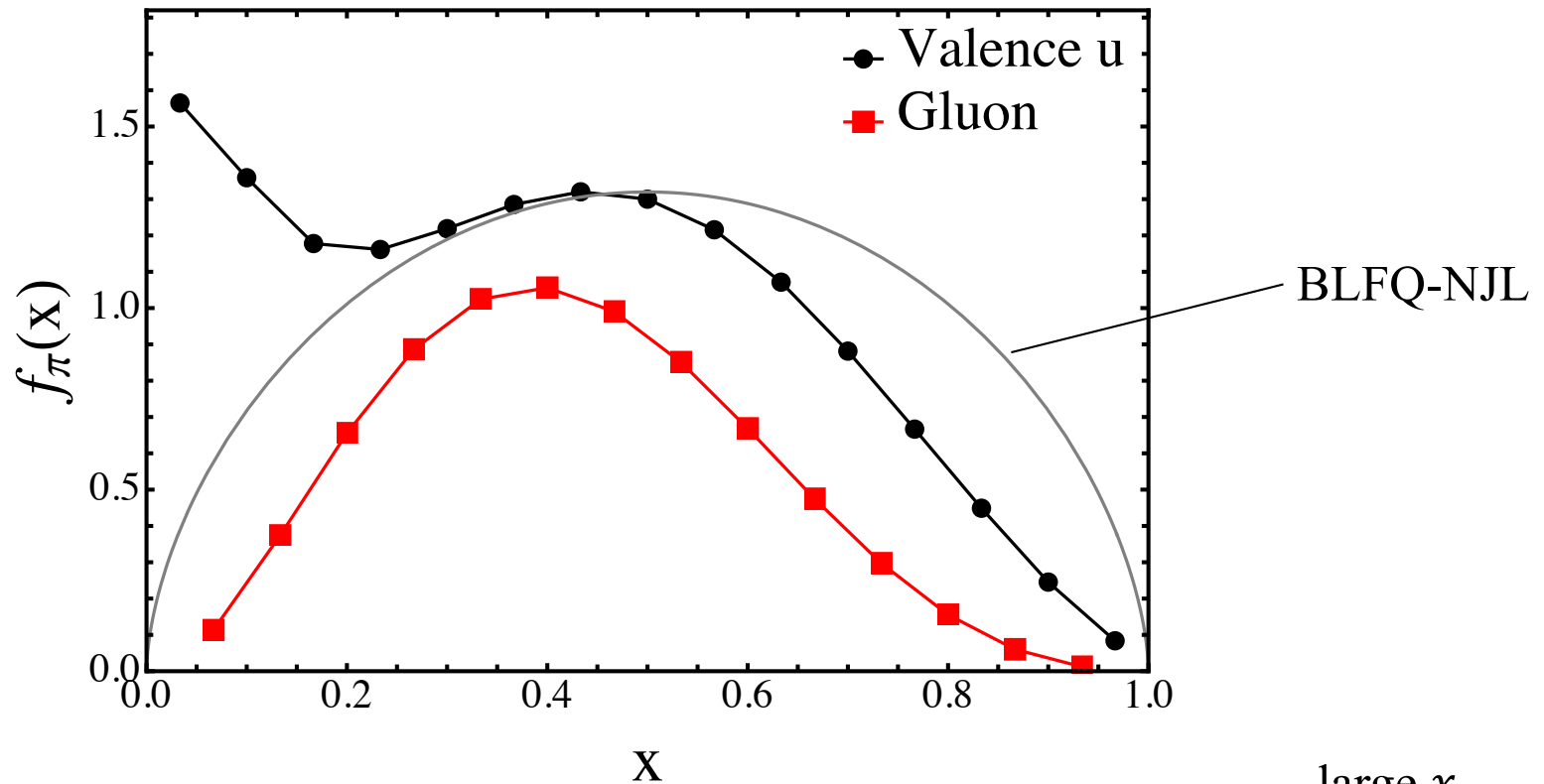


- 端点行为大致与微扰论（渐近自由极限）相符
- 与实验测量结果FNAL-E-791相符

[Jiangshan Lan, et al, in preparation]

Pion介子的部分子分布

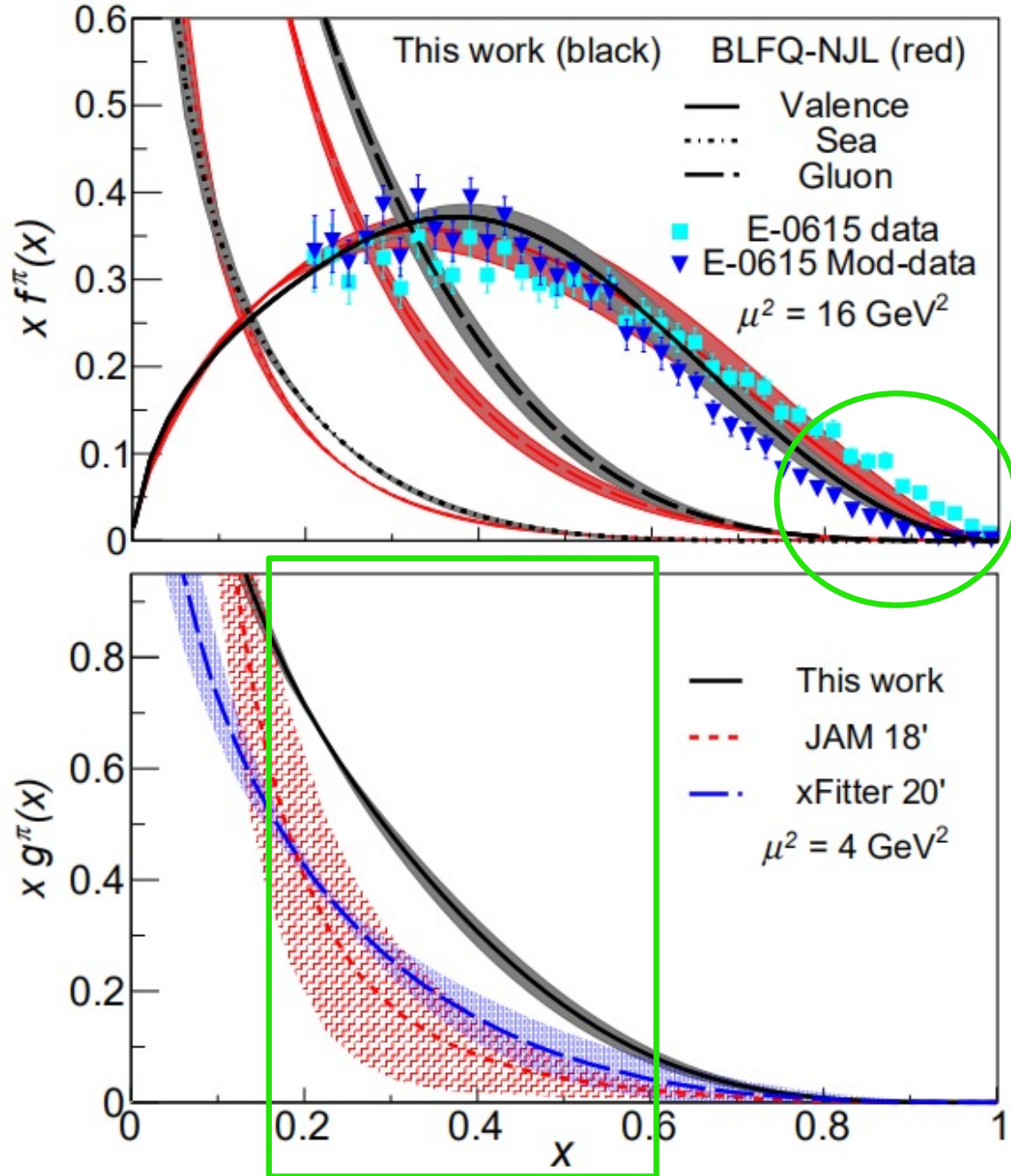
$$f_i(x) = \sum_{\mathcal{N}, \lambda_i} \int [d\mathcal{X}d\mathcal{P}^\perp]_{\mathcal{N}} \left| \psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_j=0} \right|^2 \delta(x - x_i) \quad |\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



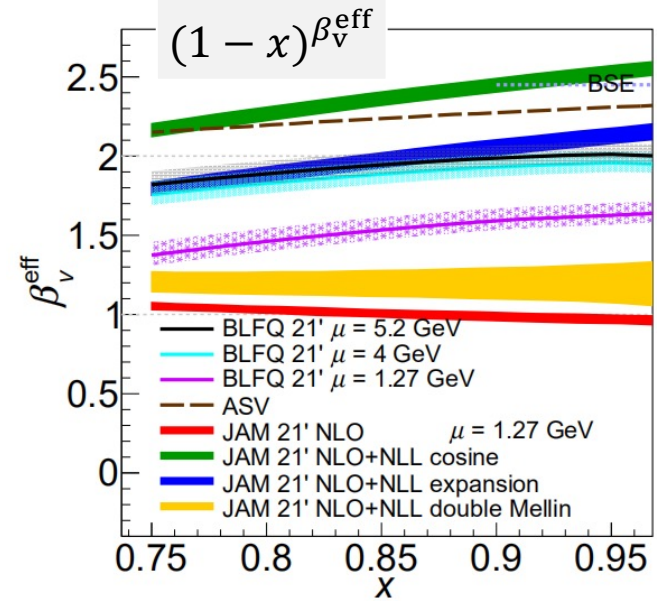
large x

$\mu_{0\text{BLFQ-NJL}}^2 = 0.240 \text{ GeV}^2$	$\langle x \rangle_{\text{gluon}} = 0;$	$\langle x \rangle_{\text{valence } u} = 0.5$	$(1-x)^{0.596}$
$\mu_{0\text{BLFQ}}^2 = 0.34 \text{ GeV}^2$	$\langle x \rangle_{\text{gluon}} = 0.216;$	$\langle x \rangle_{\text{valence } u} = 0.392$	$(1-x)^{1.4}$

Pion介子的部分子分布



$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

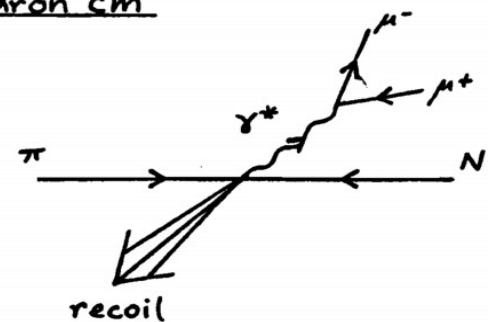


$\langle x \rangle @ 4 \text{ GeV}^2$	Valence	Gluon	Sea
BLFQ	0.483	0.421	0.096
BLFQ-NJL	0.489	0.398	0.113
[BSE 2019']	0.48(3)	0.41(2)	0.11(2)

- 大 x 的下降行为接近于微扰预测
- 胶子分布明显增加

Drell-Yan微分截面

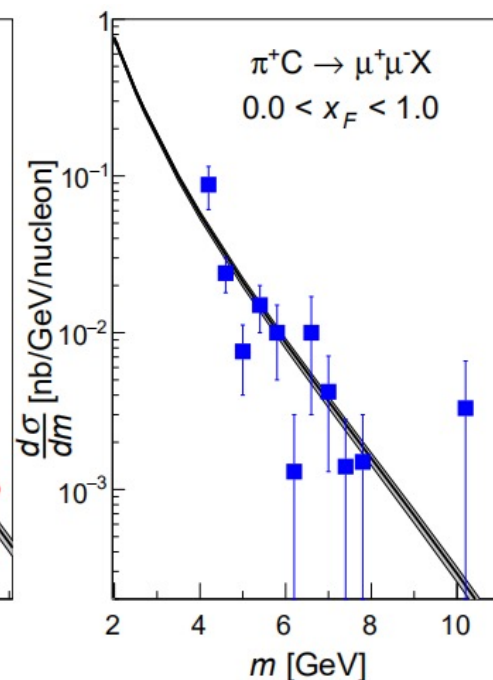
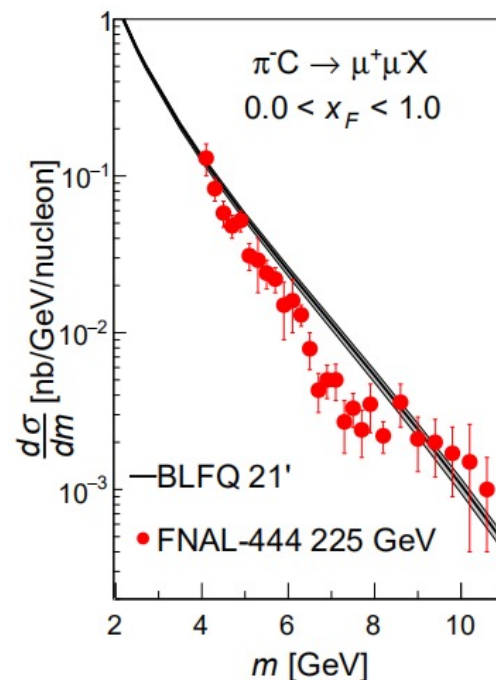
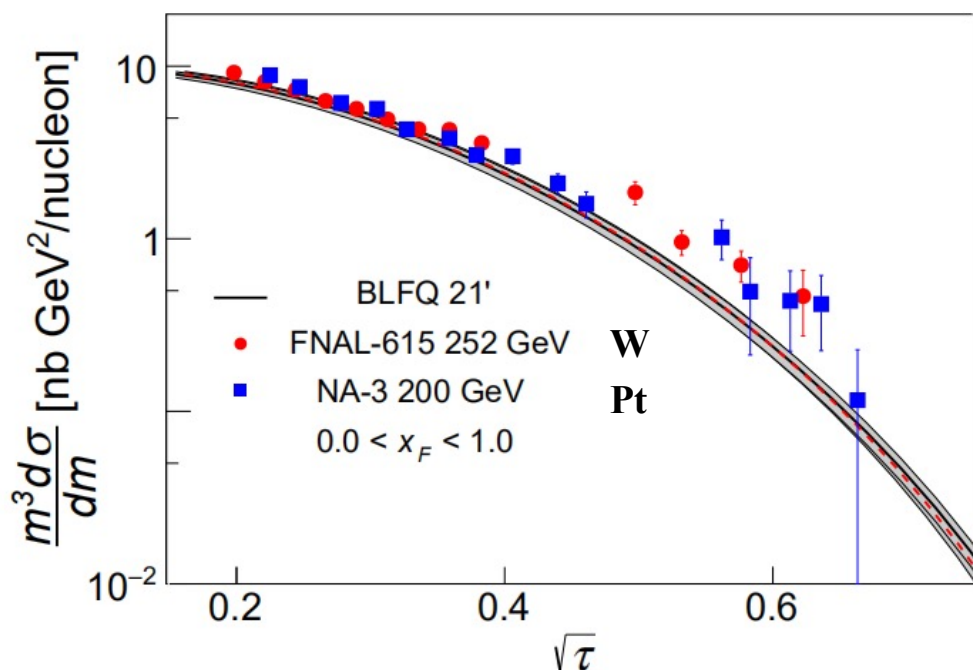
hadron cm



[S. D. Drell and T.-M. Yan, PRL (1970)];
 [Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$$\frac{m^3 d^2\sigma}{dm dY} = \frac{8\pi\alpha^2 m^2}{9 s} \sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$

[nCTEQ 2015]



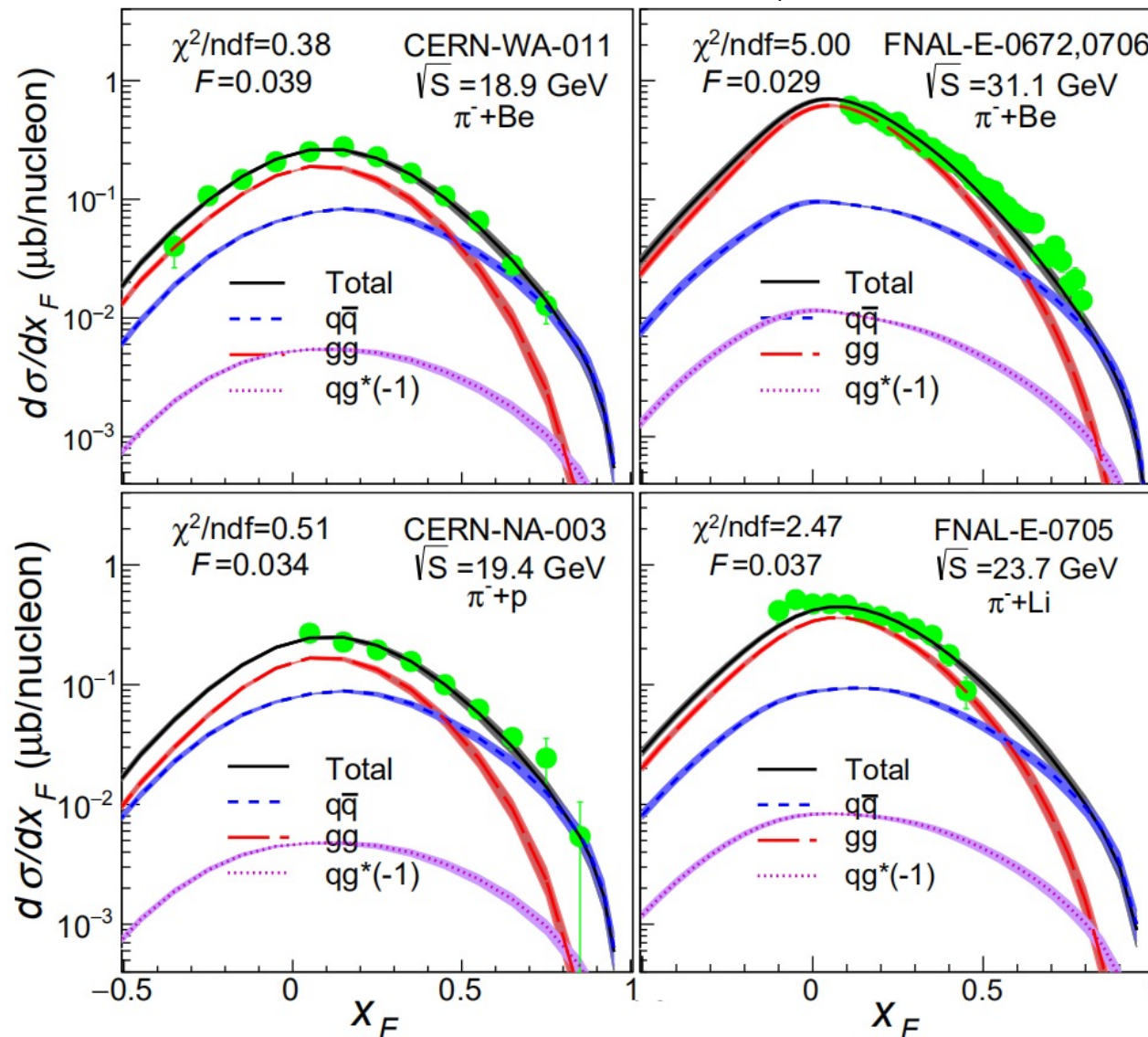
➤ 与实验结果大致相符(FNAL E615, 444 & CERN NA3).

J/ψ 产生微分截面

$$\pi^\pm N \rightarrow J/\psi X$$

$$\frac{d\sigma}{dx_F} |J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S \sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^\pm}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM

[Chang, et al, PRD 102 (2020) 054024];
 [Nason, et al, NPB 303 (1988) 607];
 [Mangano, et al, NPB 405 (1993) 507]

- 明显的gg贡献
- 不同能量的pion介子束流
- 不同原子核的靶

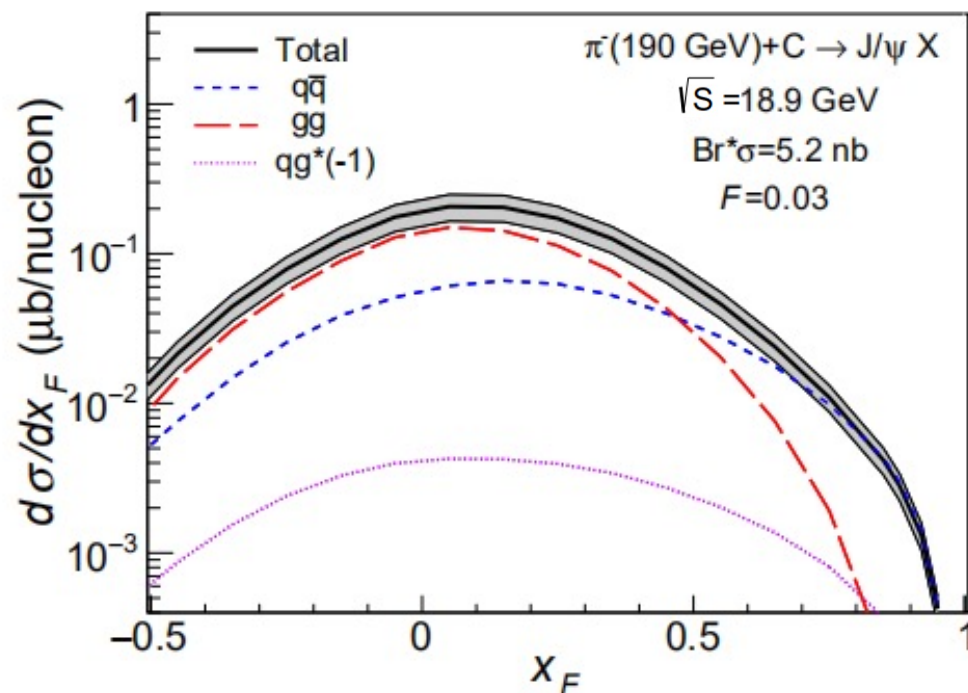
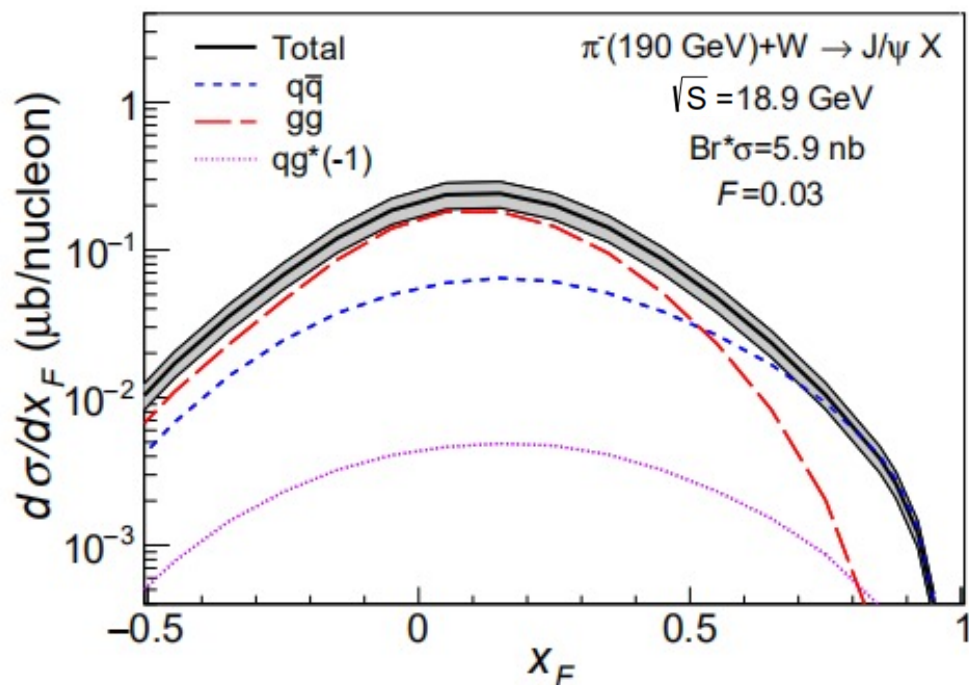
- 与实验结果大致相符
 (FNAL E672, E706, E705,
 CERN NA3, WA11).

J/ψ 产生微分截面

$$\pi^\pm N \rightarrow J/\psi X$$

$$\frac{d\sigma}{dx_F} |J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S \sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^\pm}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



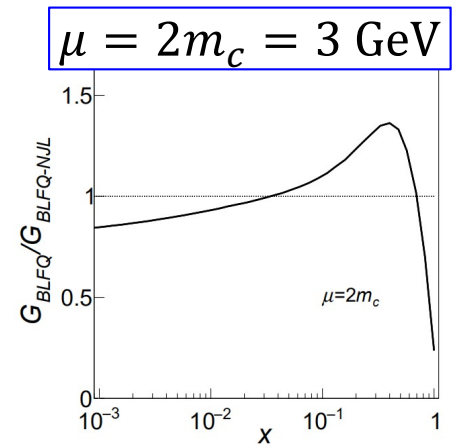
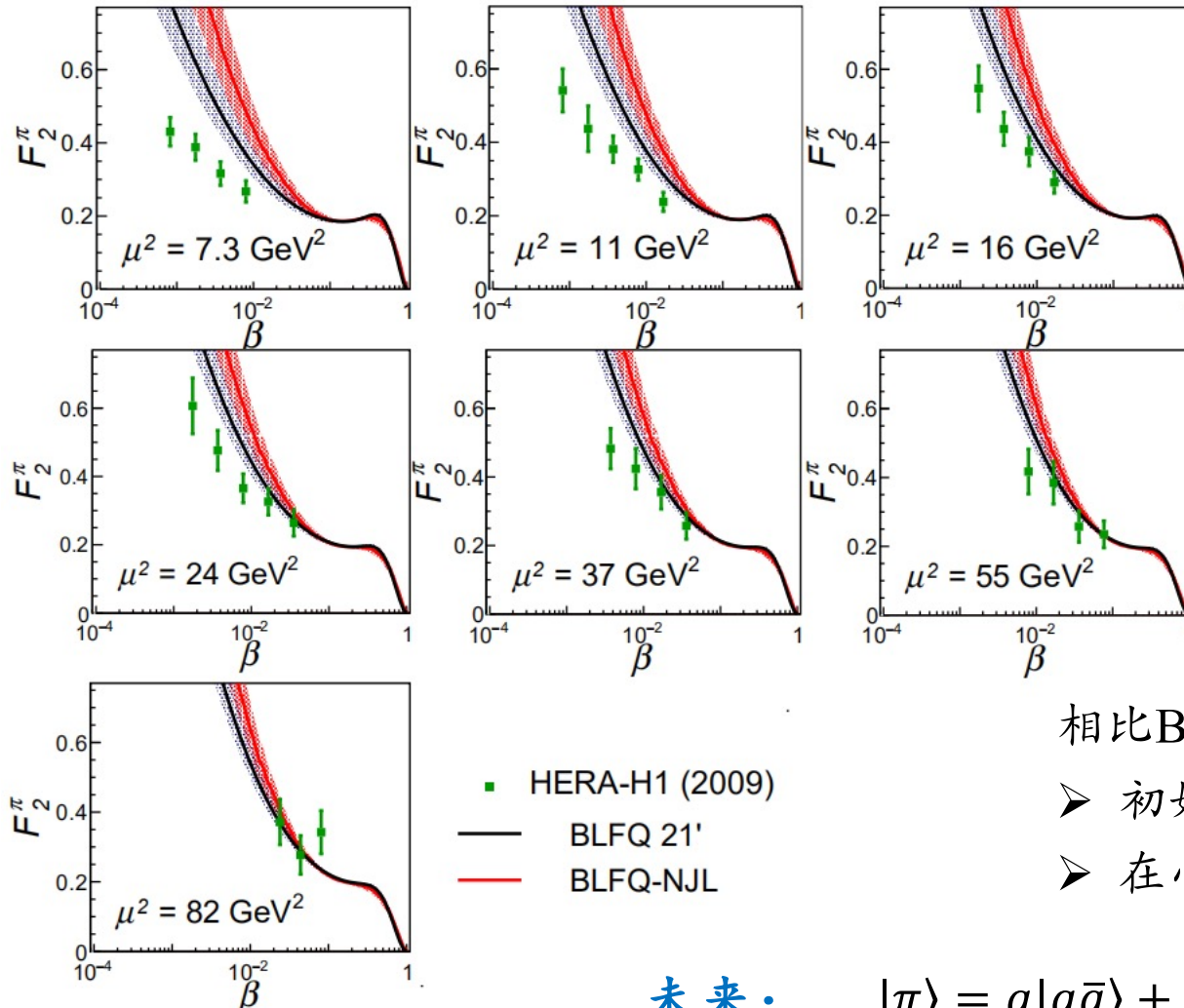
➤ COMPASS++/AMBER 将要进行的实验测量

Pion介子的结构函数

[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$F_2^\pi(\beta, \mu^2) = \sum_{q,g} e_q^2 \beta \{f_q^\pi(\beta, \mu^2) + f_{\bar{q}}^\pi(\beta, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} [C_{q,2} \otimes (f_q^\pi + f_{\bar{q}}^\pi) + 2C_{g,2} \otimes f_g^\pi]\}$$



相比BLFQ-NJL有所改善

➤ 初始能标高些

➤ 在小x区域，胶子和海夸克更少

[Jiangshan Lan, et al, in preparation]

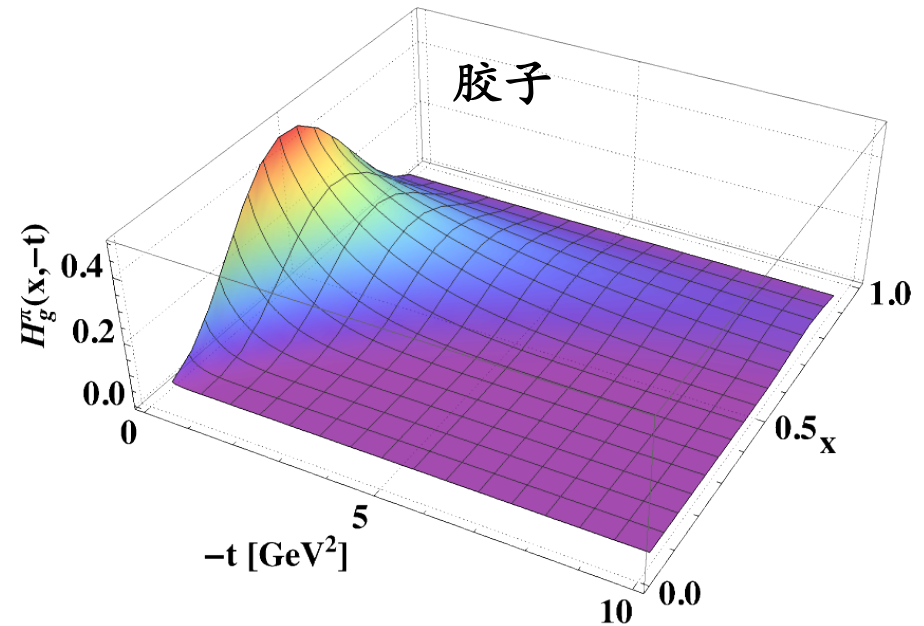
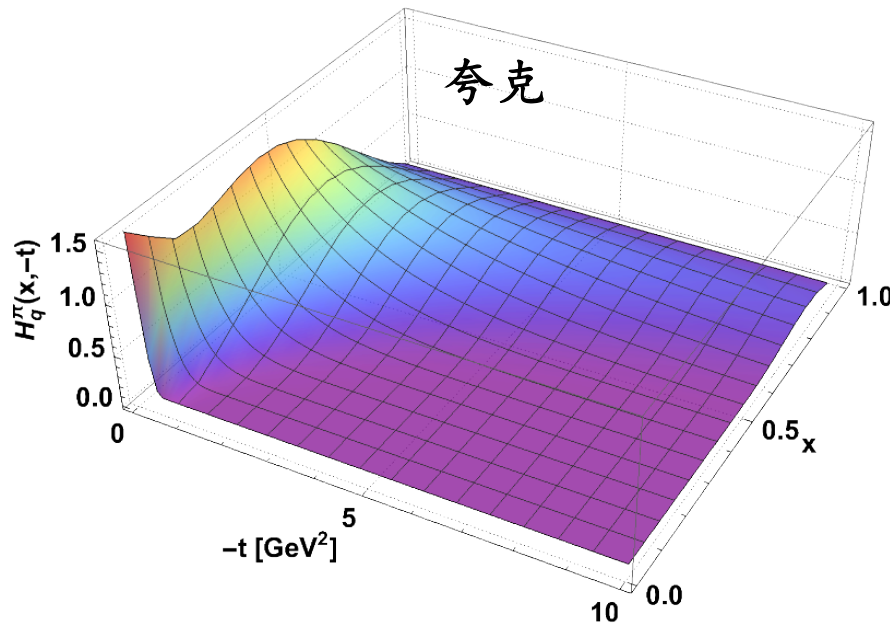
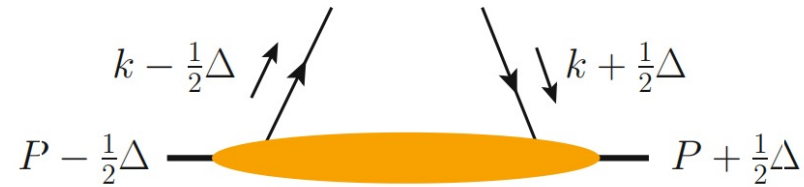
未来：

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + \dots \quad 19$$

Pion介子的广义部分子分布

[M. Diehl, Phys. Rep. 388 (2003) 41-277]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

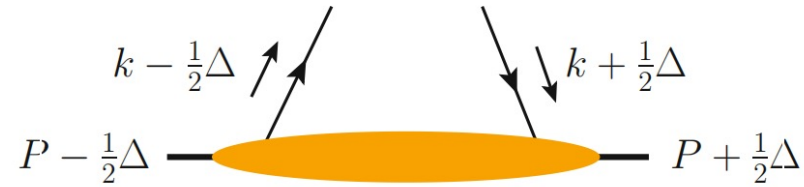


- 由于 $|q\bar{q}g\rangle$ 的贡献，夸克在小 x 区域有增强
- 在大 x 区域下降比较慢
- 对于大的 $-t$ ，只分布在大 x 区域

Pion介子的广义部分子分布

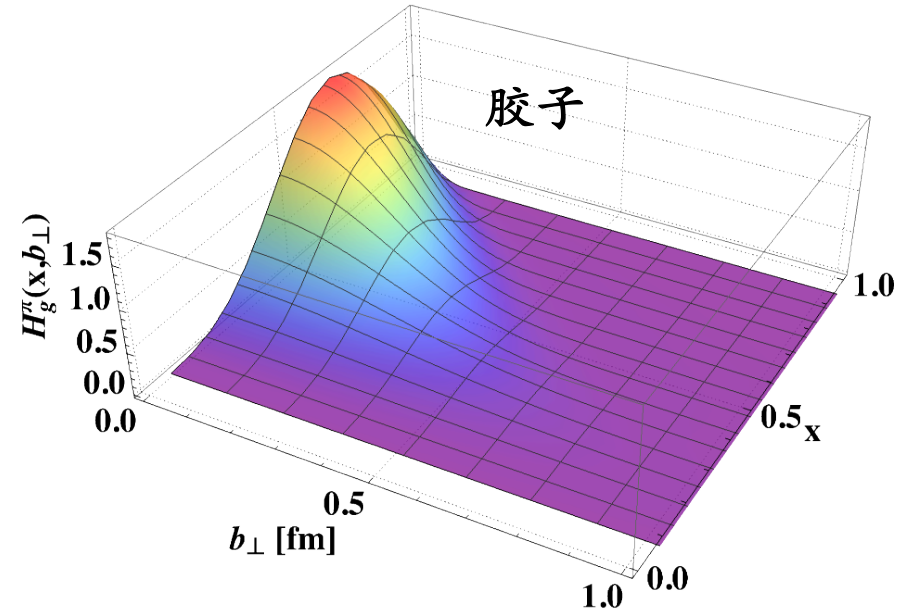
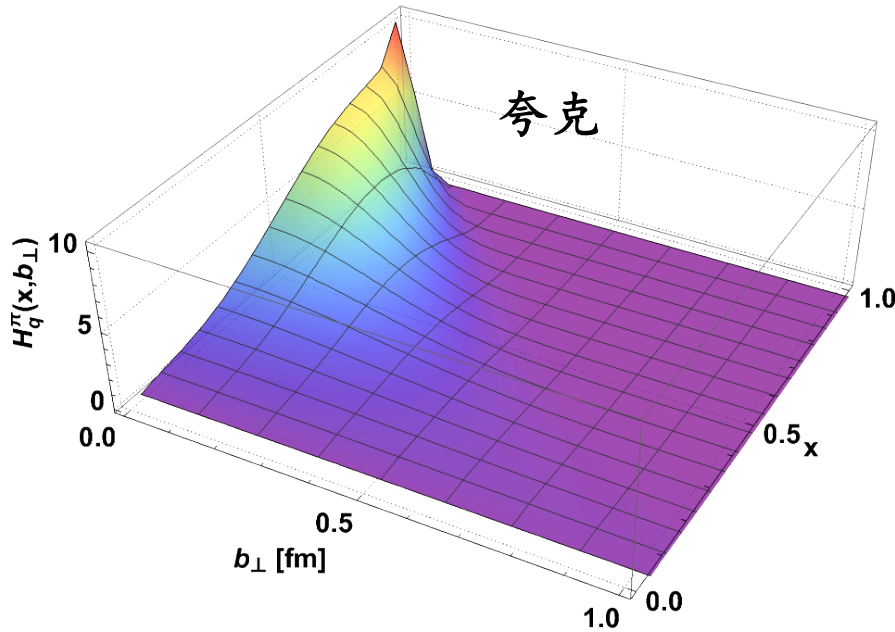
[M. Diehl, Phys. Rep. 388 (2003) 41-277]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



傅里叶变换

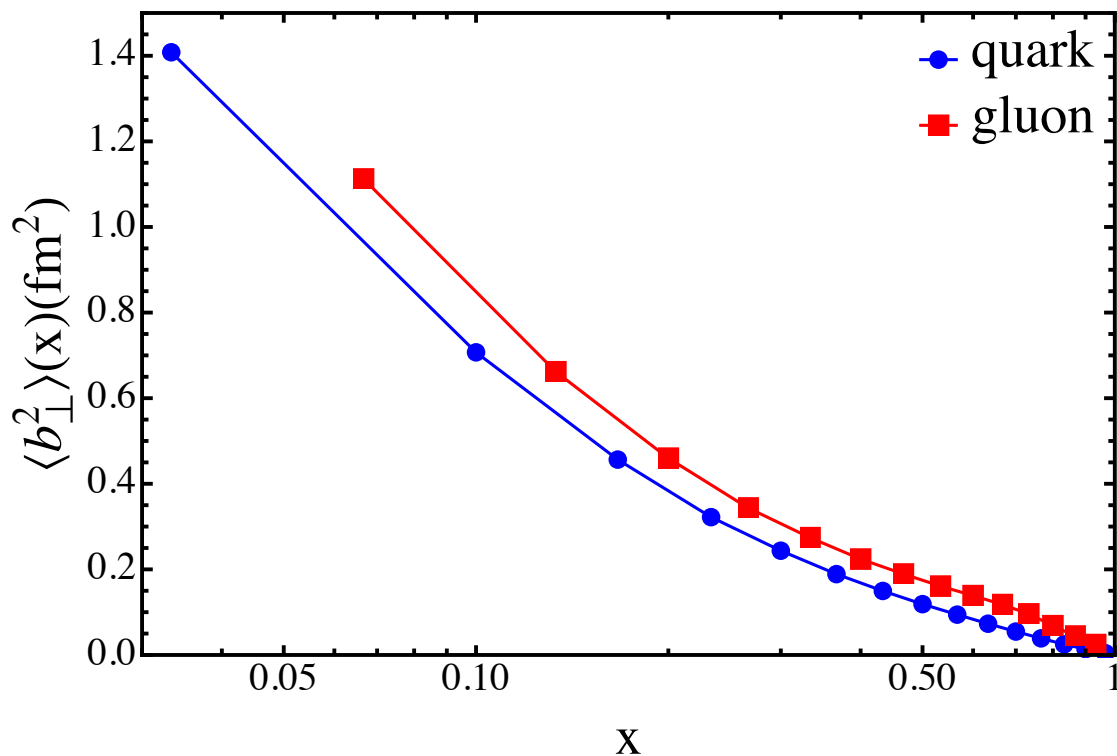
碰撞参量分布 The impact parameter distributions (IPDs)



Pion介子的碰撞参量分布

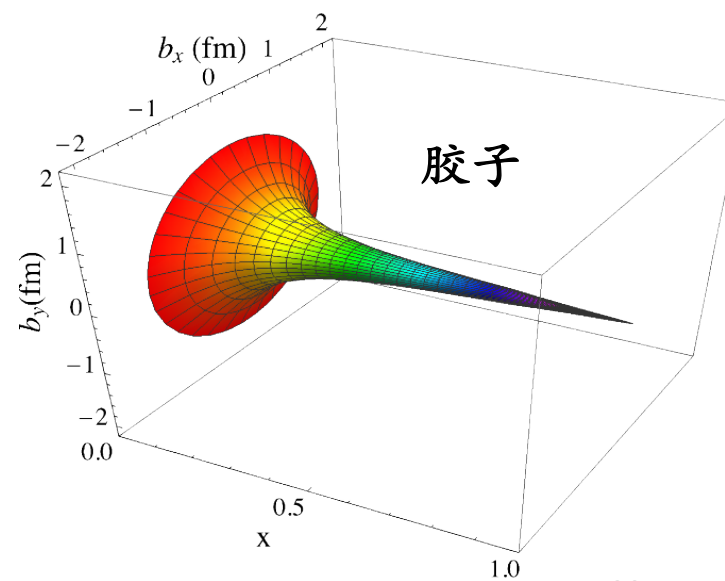
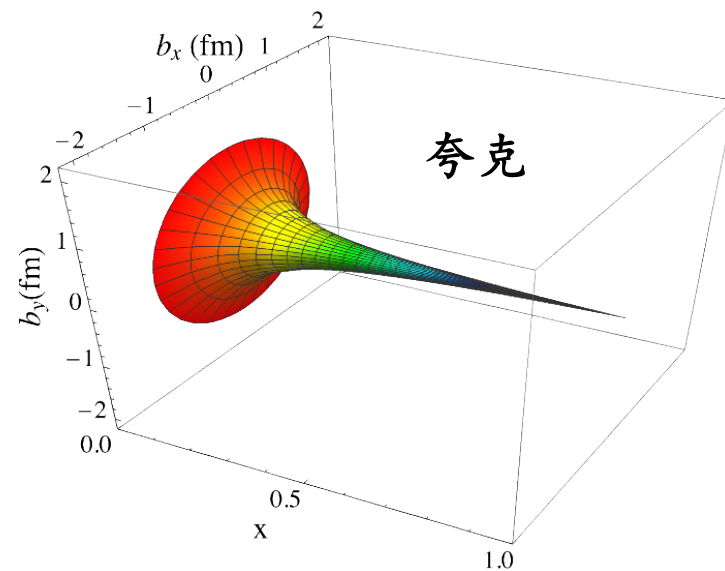
如果我们定义一个 x 依赖的半径来描述夸克和胶子在横向上的密度分布:

$$\langle b_{\perp}^2 \rangle^{q,g}(x) = \frac{\int d^2 \mathbf{b}_{\perp} b_{\perp}^2 H_{\pi}^{q,g}(x, \mathbf{b}_{\perp})}{\int d^2 \mathbf{b}_{\perp} H_{\pi}^{q,g}(x, \mathbf{b}_{\perp})}$$



➤ 胶子分布比夸克分布稍微宽一些

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

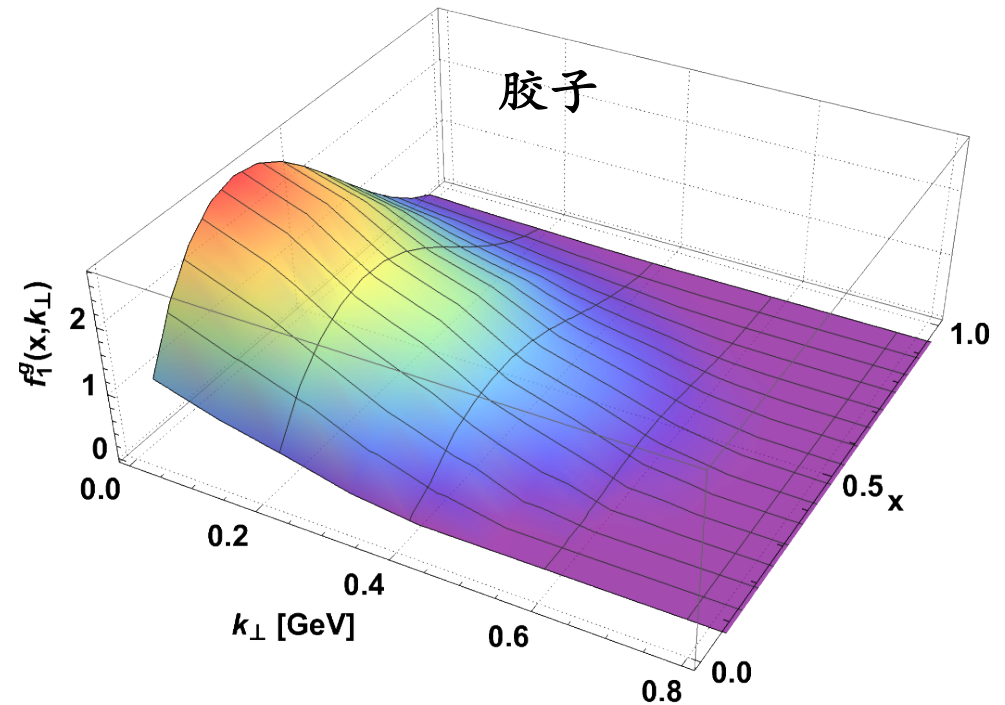
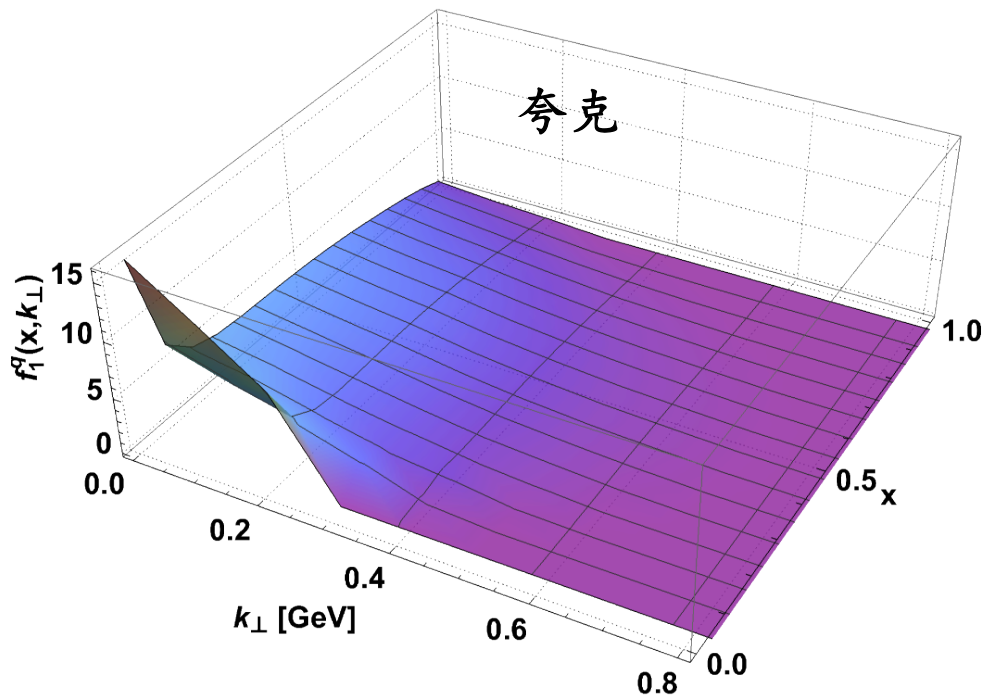


Pion介子的横向动量依赖部分子分布

[Boer & Mulders PRD 57 (1998) 5780]

[Pasquini et al, PRD 90 (2014) 014050]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



- 分布随 k_{\perp} 增加而减少
- 在 $k_{\perp} \sim 0.7$ GeV 后, 消失

[Jiangshan Lan, et al, in preparation]

总结与展望

- 光前哈密顿量方法：**介子能谱** \longleftrightarrow **介子结构**
 - 在包含一个动力学胶子的轻介子中，
 - ✓ 可描述 π 介子的电磁形状因子，并给出大 Q^2 的行为 $F(Q^2) \propto 1/Q^2$
 - ✓ 可描述 π 介子的部分子分布振幅和部分子分布
 - ✓ π 介子中胶子分布在 $0.2 < x < 0.6$ 区域增多，而在小 x 区域压低
 - ✓ 可描述Drell-Yan散射微分截面和 J/ψ 产生微分截面
 - 可以用来研究三维结构包括**广义部分子分布**和**横向动量依赖部分子分布**
-

- 逐渐包含更高Fock态

$$|\text{Meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|gg\rangle + e|q\bar{q}gg\rangle + \dots$$

- 除 π 介子以外的其他介子 (K 、 η 、 ρ 介子等轻介子&重介子)

谢谢！ 请各位专家批评指正！