中国科学院近代物理研究所
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Neutrino-^{12}C & ^{13}C scattering in *No-Core Configuration Interaction*

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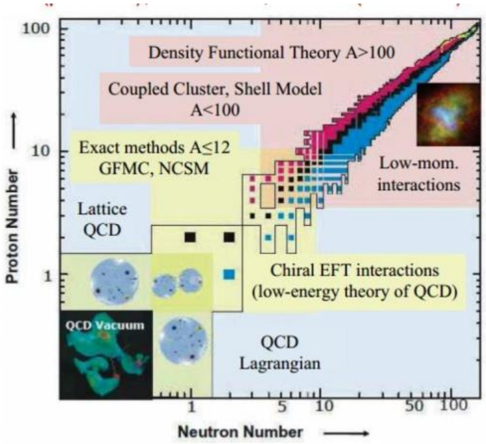
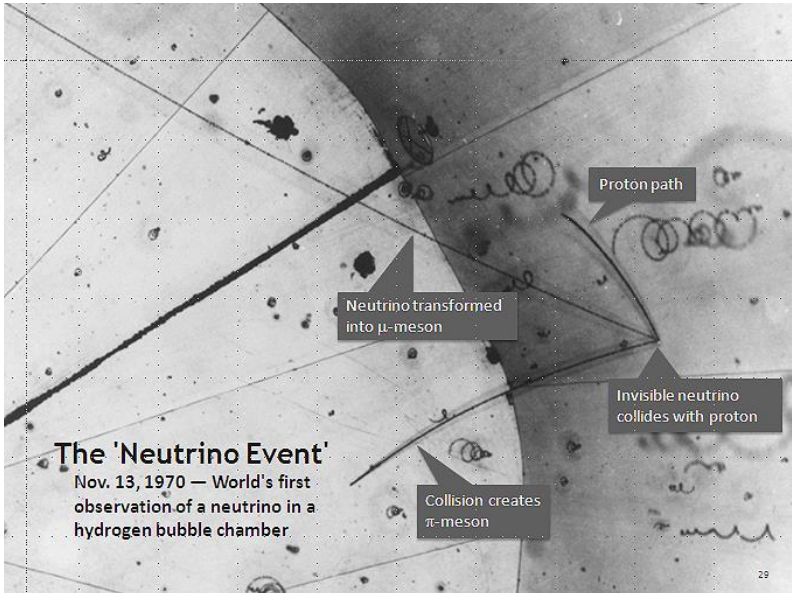
Outline

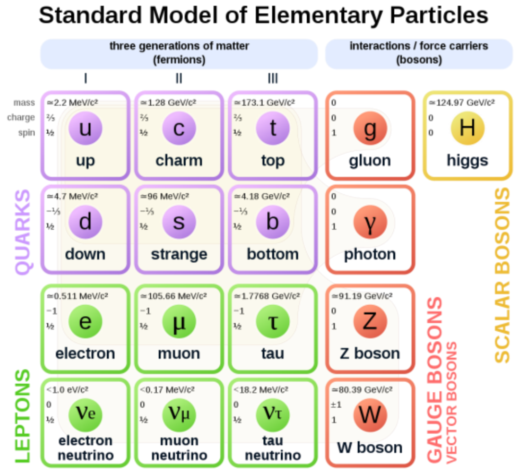
- ➢ **Neutrino-C scattering in Jiangmen Underground Neutrino Observatory (JUNO)**
- ➢ **No-core configuration interaction (NCCI) method**
- ➢ **Daejeon16 interaction**
- ➢ **Current progress @ Sugon supercomputer**
- ➢ **Next plan and conclusions**





Main Physics goal in JUNO

❖ Determination of the Neutrino Mass Ordering

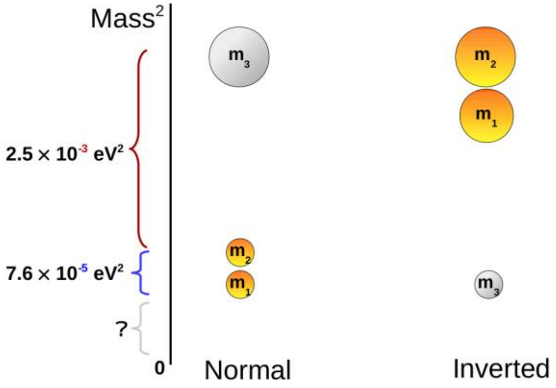


Flavor eigenstates

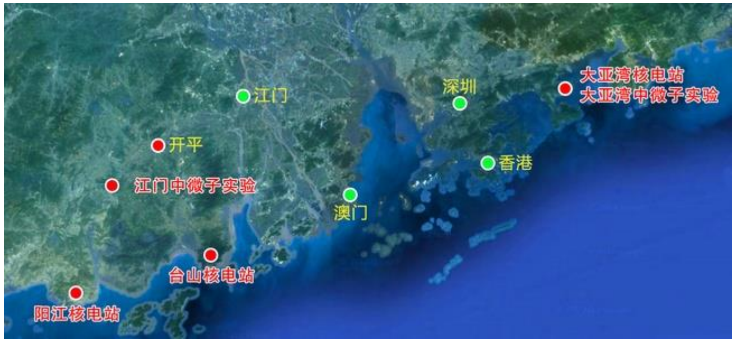
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMSN matrix

m1<m2<m3 m3<m1<m2



Mass eigenstates



Reactor neutrinos
from Taishan & Yangjiang

Positive/Negative?

$$\Delta m^2_{21} = 7.54\left(1.00^{+0.034}_{-0.029}\right)\times 10^{-5}\ \text{eV}^2,$$

$$\sin^2\theta_{12} = 0.308\left(1\pm 0.055\right),$$

$$\left|\Delta m^2_{32}\right| = 2.39\left(1\pm 0.025\right)\times 10^{-3}\ \text{eV}^2,$$

$$\sin^2\theta_{23} = 0.437\left(1.00^{+0.076}_{-0.053}\right),$$

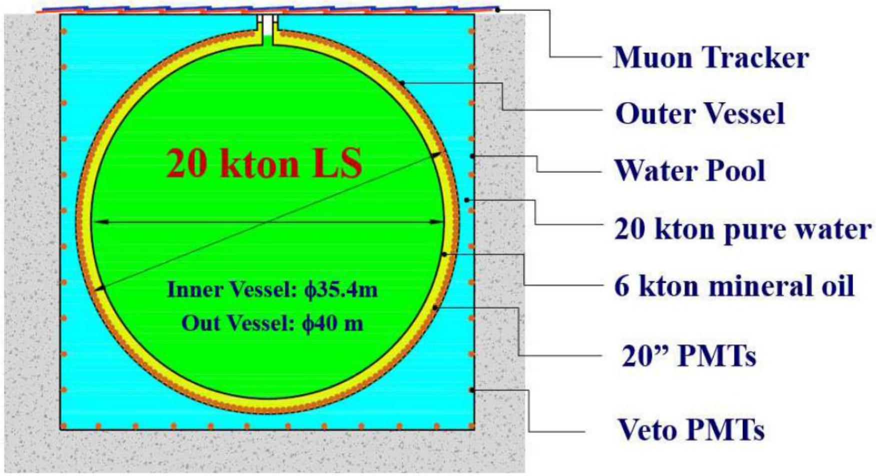
$$\sin^2\theta_{13} = 0.0234\left(1.00^{+0.085}_{-0.091}\right).$$

Neutrino oscillation parameters
Assume (m1<m2<m3)

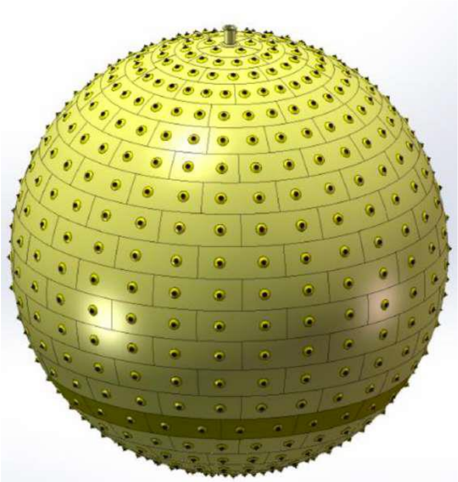
曹俊.中国科学:物理学 力学 天文学(2014)

Main Physics goal in JUNO

❖ Mass Ordering Determination – Experiments: Reactor Neutrino Events



JUNO Detector Concept



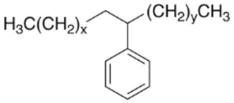
JUNO Central Detector

Channel	Type	Events for different $\langle E_\nu \rangle$ values		
		12 MeV	14 MeV	16 MeV
$\overline{\nu}_e + p \to e^+ + n$	CC	4.3×10^3	5.0×10^3	5.7×10^3
$\nu + p \to \nu + p$	NC	0.6×10^3	1.2×10^3	2.0×10^3
$\nu + e \to \nu + e$	ES	3.6×10^2	3.6×10^2	3.6×10^2
$\nu + {}^{12}\mathrm{C} \to \nu + {}^{12}\mathrm{C}^*$	NC	1.7×10^2	3.2×10^2	5.2×10^2
$\nu_e + {}^{12}\mathrm{C} \to e^- + {}^{12}\mathrm{N}$	CC	0.5×10^2	0.9×10^2	1.6×10^2
$\overline{\nu}_e + {}^{12}\mathrm{C} \to e^+ + {}^{12}\mathrm{B}$	CC	0.6×10^2	1.1×10^2	1.6×10^2

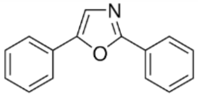
Numbers of neutrino events in JUNO for a Supernova at a typical distance of 10 kpc(about 3.26×10^4 light-years)

Composition: 20 kt LS : 3g/L PPO +15 mg/L bisbis-MSB in LAB

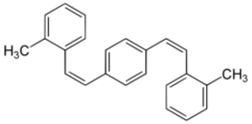
- LAB



- PPO (3 g/l)



- bis-MSB (15 mg/l)

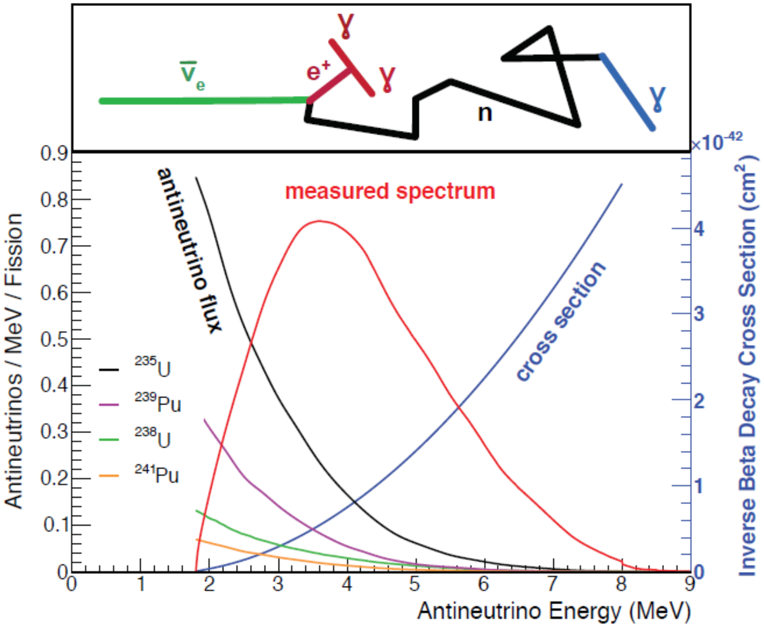


Fengpeng An et al., J. Phys. G 43 (2016)

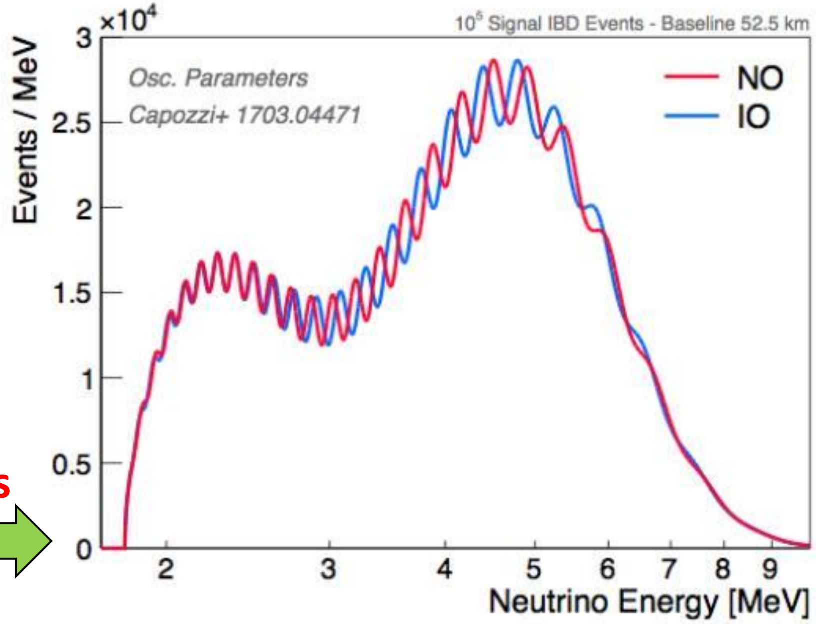
Wenqiang Gu, Status and Prospects of JUNO Shang Hai Jiao Tong University (2019)

Main Physics goal in JUNO

- **Mass Ordering Determination –Theoretical Calculations: cross-sections**



High Precision Cross Section Calculations

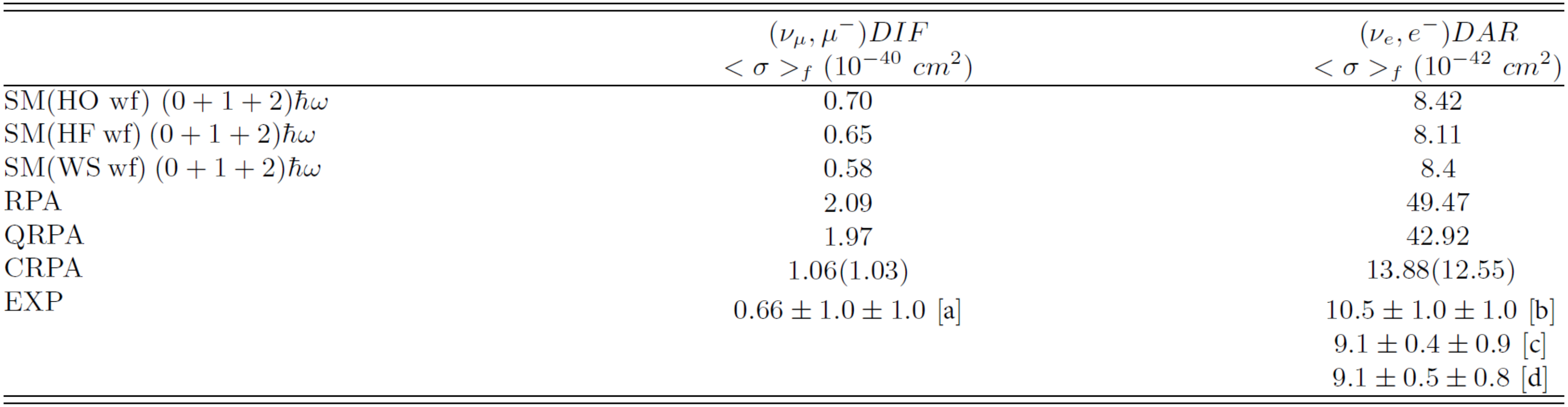


Neutrino events + Neutrino-nuclei scattering cross-sections → Neutrino energy spectra
From detector | **From theoretical calculations** | **Mass ordering**

Fengpeng An et al., J. Phys. G 43 (2016)

Phenomenological models calculations

❖ Neutrinos-^{12}C cross-sections calculations with phenomenological models



	$(\nu_\mu, \mu^-)DIF$ $<\sigma>_f$ $(10^{-40}\ cm^2)$	$(\nu_e, e^-)DAR$ $<\sigma>_f$ $(10^{-42}\ cm^2)$
SM(HO wf) $(0+1+2)\hbar\omega$	0.70	8.42
SM(HF wf) $(0+1+2)\hbar\omega$	0.65	8.11
SM(WS wf) $(0+1+2)\hbar\omega$	0.58	8.4
RPA	2.09	49.47
QRPA	1.97	42.92
CRPA	1.06(1.03)	13.88(12.55)
EXP	$0.66 \pm 1.0 \pm 1.0$ [a]	$10.5 \pm 1.0 \pm 1.0$ [b] $9.1 \pm 0.4 \pm 0.9$ [c] $9.1 \pm 0.5 \pm 0.8$ [d]

Neutrinos-^{12}C cross-sections from different models
Shell Model (SM)
Random Phase Approximation (RPA)
Quasi-particle RPA (QRPA)
Continuum RPA (CRPA)

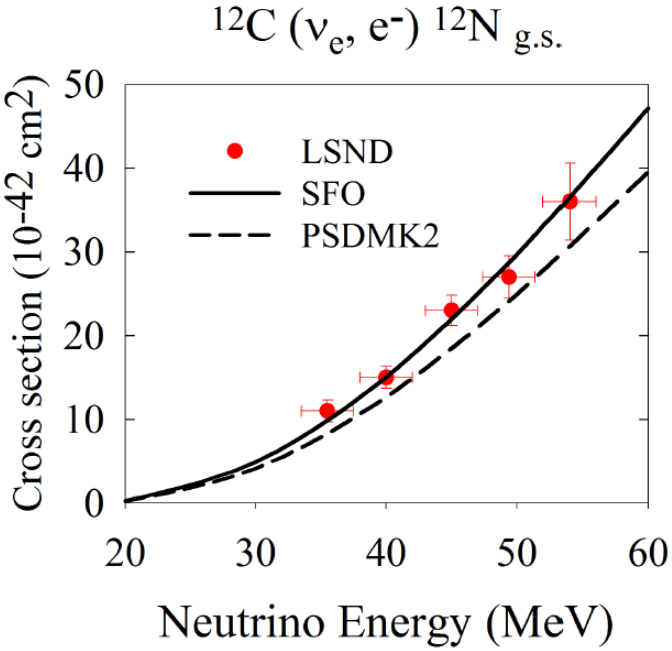
a: C.Athanassopoulos and the LSND collaboration, Phys. Rev. C (1997),
b: R.C.Allen et al., Phys. Rev. Lett (1990),
c: C.Athanassopoulos and the LSND collaboration, Phys. Rev. C (1997),
d: B.E.Bodmann and the KARMEN collaboration, Phys. Lett. B (1994)

The results from RPA calculations are far away from experimental data, while the SM calculations are much more close to experimental data

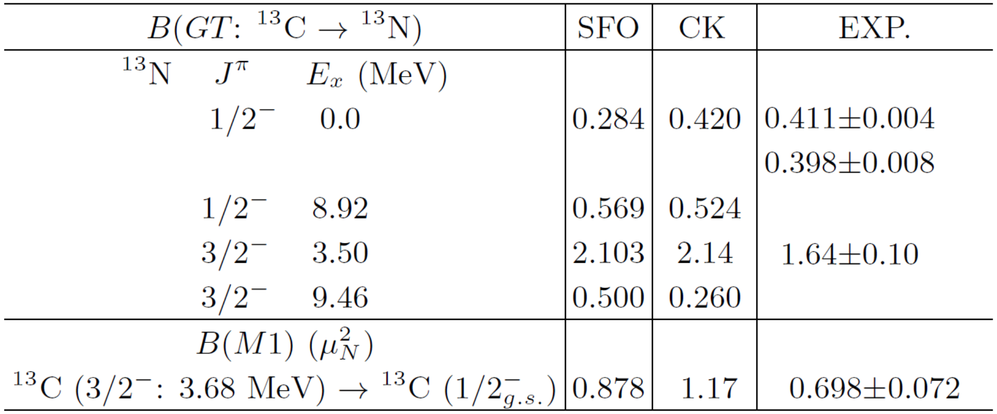
Volpe et al., Phys. Rev. C (2000)

Phenomenological models calculations

- **Neutrino-^{12}C & ^{13}C cross-sections calculations with shell model**



Neutrino-^{12}C charge current cross-sections from shell model with SFO interaction had a good agreement with experimental data



$B(GT:\ ^{13}\mathrm{C} \to\ ^{13}\mathrm{N})$		SFO	CK	EXP.
^{13}N J^π	E_x (MeV)			
$1/2^-$	0.0	0.284	0.420	0.411±0.004
				0.398±0.008
$1/2^-$	8.92	0.569	0.524	
$3/2^-$	3.50	2.103	2.14	1.64±0.10
$3/2^-$	9.46	0.500	0.260	
$B(M1)$ (μ_N^2)				
$^{13}\mathrm{C}\ (3/2^-\!:\ 3.68\ \mathrm{MeV}) \to\ ^{13}\mathrm{C}\ (1/2^-_{g.s.})$		0.878	1.17	0.698±0.072

B(M1) and B(GT) strengths of ^{13}C from shell model with SFO & CK interaction were still a little far away from experimental data

T. Suzuki et al., Phys. Rev. C (2012)
T. Suzuki et al., Phys. Rev. C (2019)

Introduction of NCCI/NCSM

❖ No-Core Configuration Interaction / No-Core Shell Model calculations

- Given a Hamiltonian operator

$$\hat{H} = \sum_{i<j} \frac{(\vec{p}_i - \vec{p}_j)}{2mA} + \sum_{i<j}^{A} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$$

- Solve the eigenvalue problem for wavefunction of A nucleons

$$\hat{H}\Psi_{\alpha_1 \cdots \alpha_A} = \mathrm{E}\Psi_{\alpha_1 \cdots \alpha_A}$$

- Expand wavefunction in basis states $|\psi\rangle = \sum \hat{a}_j \, |\phi_i\rangle$
- Express Hamiltonian in basis $\langle \phi_j | \hat{H} | \phi_i \rangle = H_{ij}$
- Diagonalize Hamiltonian matrix H_{ij}
- No-Core: All A nucleons are treated the same
- Complete basis ---- exact result
- In practice
  1. truncate basis
  2. study behavior of observables as function of truncation
- Computational challenge
  1. construct large ($10^{10} \times 10^{10}$) sparse symmetric matrix H_{ij}
  2. obtain lowest eigenvalues & eigenvectors corresponding to low-lying spectrum and eigenstates

For application to hadron structure see Jiangshan Lan's talk on BLFQ (Monday afternoon)

P. Maris, NCSM_NICC_YTT(2021)

Barrett et al., Progress in Particle and Nuclear Physics (2013)

Introduction of NCCI/NCSM

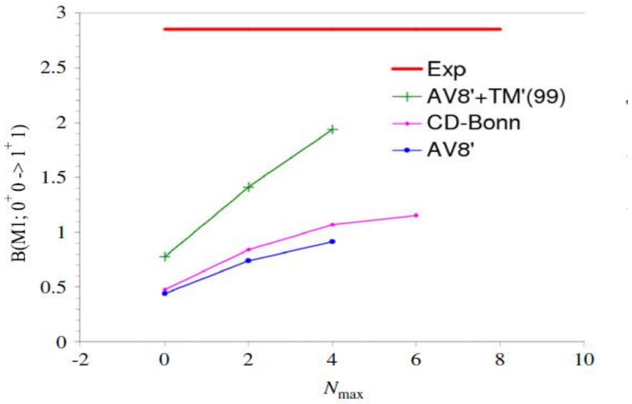
- **MFDn (Many Fermion Dynamics for Nuclear Structure) code**

  - **Calculate the approximate low-lying spectrum (eigenvalues) and corresponding wavefunctions ψ (eigenvectors) of a (Z + N) = A-body system of Z protons and N neutrons using 2-body (and optional 3-body) interactions in a finite basis in configuration space (sparse matrix)**
  - **Evaluate for the requested number of states (eigenvalues)**
    1. **orbital occupation probabilities**
    2. **One-Body Density Matrix Elements (OBDMEs)**
    3. **expectation values of total spin J^2 and isospin T^2**
       (for T^2 assuming proton and neutron radial wavefunctions are the same)
    4. **expectation values for arbitrary scalar 2-body operators**
    5. **for Harmonic Oscillator (HO) basis**
       static: quadrupole moments, M > 0: magnetic moments, r^2 …
       transitions: M1, E2, and GT(assuming isospin symmetry)

P. Maris, NCSM_NICC_YTT(2021)

Previous calculations from Ab initio NCSM

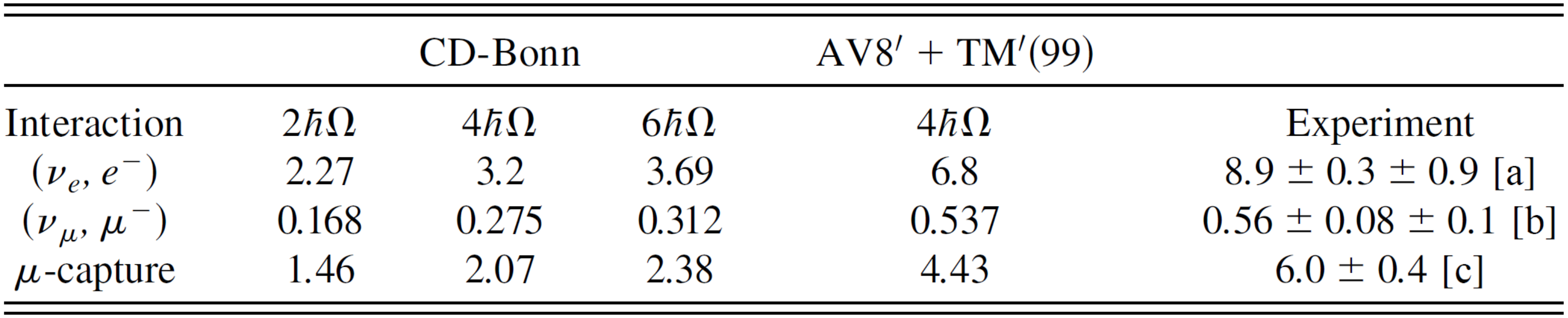
❖ Neutrino-^{12}C cross-sections calculations with NCSM



AV8’: NN potential
CD-Bonn: NN potential
AV8’ + TM’(99):
NN potential + 3N NN potential

The B(M1) calculated values from AV8’, CD-Bonn, AV8’ + TM’(99) were still a little far away from experimental data

Experimental and calculated B(M1;0$^+$0 → 1$^+$1) values for ^{12}C



	CD-Bonn			AV8$'$ + TM$'$(99)	
Interaction	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	$4\hbar\Omega$	Experiment
(ν_e, e^-)	2.27	3.2	3.69	6.8	$8.9 \pm 0.3 \pm 0.9$ [a]
(ν_μ, μ^-)	0.168	0.275	0.312	0.537	$0.56 \pm 0.08 \pm 0.1$ [b]
μ-capture	1.46	2.07	2.38	4.43	6.0 ± 0.4 [c]

Predicted weak interaction rates of ^{12}C. The units are 10^{-42} cm^2 for the (ν_e, e^-) DAR cross section, 10^{-42} cm^2 for the (ν_μ, μ^-) DIF cross section, and 10^3 sec^{-1} for muon capture

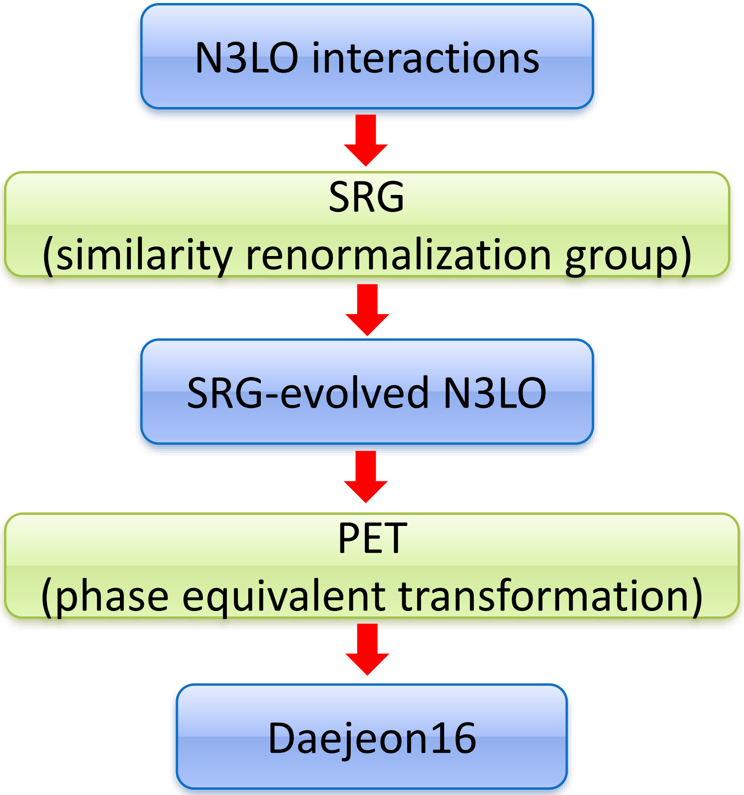
a:LSND Collaboration, L. B. Auerbach et al., Phys. Rev. C 64, 065501 (2001).
b:LSND Collaboration, L. B. Auerbach et al., Phys. Rev. C 66, 015501 (2002).
c: G. H. Miller et al., Phys. Lett. B 41, 50 (1972);

Hayes et al., Phys. Rev. Lett (2003)

Barrett et al., , Progress in Particle and Nuclear Physics (2013)

Daejeon16 interaction

- **Daejeon16 (Daejeon, the name of a city in South Korea, where the main developments took place. PET tuned up to ^{16}O)**
  based on chiral N3LO NN potential



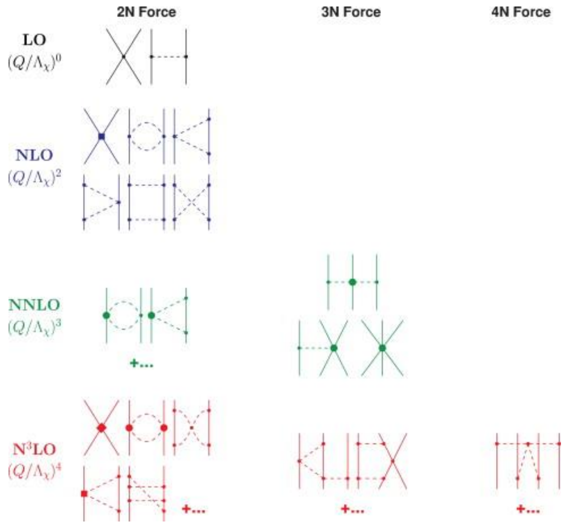
N3LO interactions

SRG
(similarity renormalization group)

SRG-evolved N3LO

PET
(phase equivalent transformation)

Daejeon16



PET angles (in degrees) defining the Daejeon16 NN interaction in various NN partial waves.

Wave	${}^1s_0$	${}^3sd_1$	${}^1p_1$	${}^3p_0$	${}^3p_1$	${}^3pf_2$	${}^3d_2$
Angle	−2.997	4.461	5.507	1.785	4.299	−2.031	7.833

Kim et al., arXiv:1910.04367 (2019)
Machleidt et al., Phys. Rept. (2011)
Shirokov et al., Physics Letters B(2016)

Improvements of our calculations

From the previous NCSM calculations of neutrino-^{12}C scattering, we can infer that the precision of neutrino-^{12}C cross-sections with Ab initio NCSM/NCCI, could be improved by applying new interaction(with the contributions from 3N force), increasing the basis space, including more reaction channels and excited states.

❖ Highlights in our calculations

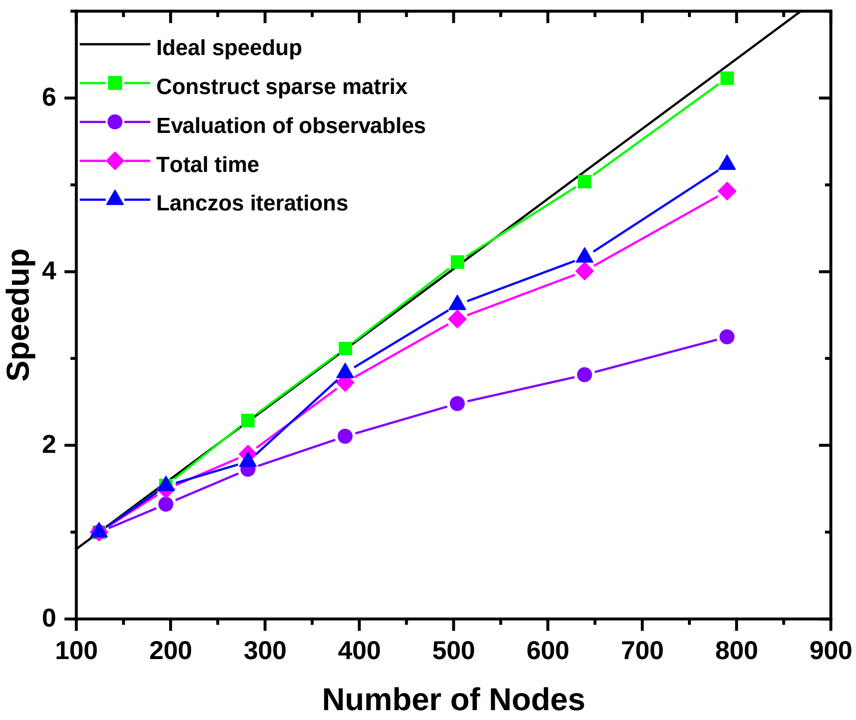
1. **New interaction – Daejeon16**
2. **Larger basis space for calculations(up to Nmax=10)**
3. **Higher excitation energy (at least 20 MeV~ 30 states)**
4. **More channels(neutral current, charged current, nucleon knock-out)**
5. **Including neutrino-^{13}C scattering(natural abundance of ^{13}C is 1.07%)**
6. **Under different hw values, to check the convergence of calculated results**
7. **Cross sections as function of neutrino energy at reactor neutrino energies**

Dimensions of many body matrix in Nmax6: $3.26\text{x}10^7$; Nmax10: $7.83\text{x}10^9$

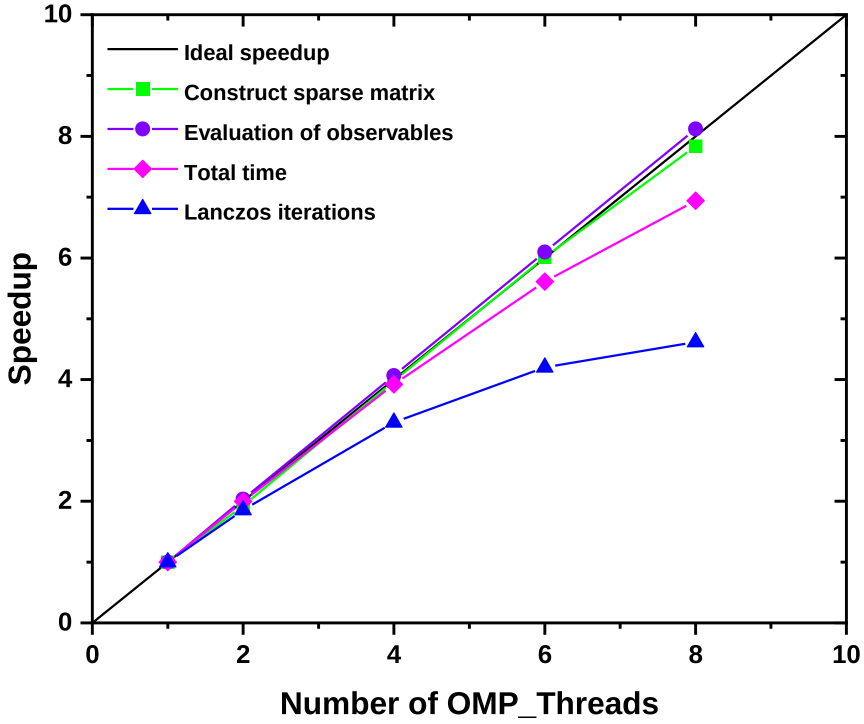
Sugon high performance computer

- **Hygon CPU node: 32 cores, 128Gb memory**
- **Kunshan center: more than 10,000 nodes**
- **Taiyuan center: ~ 7000 nodes**

❖ Scaling test of MFDn code on Sugon high performance computers



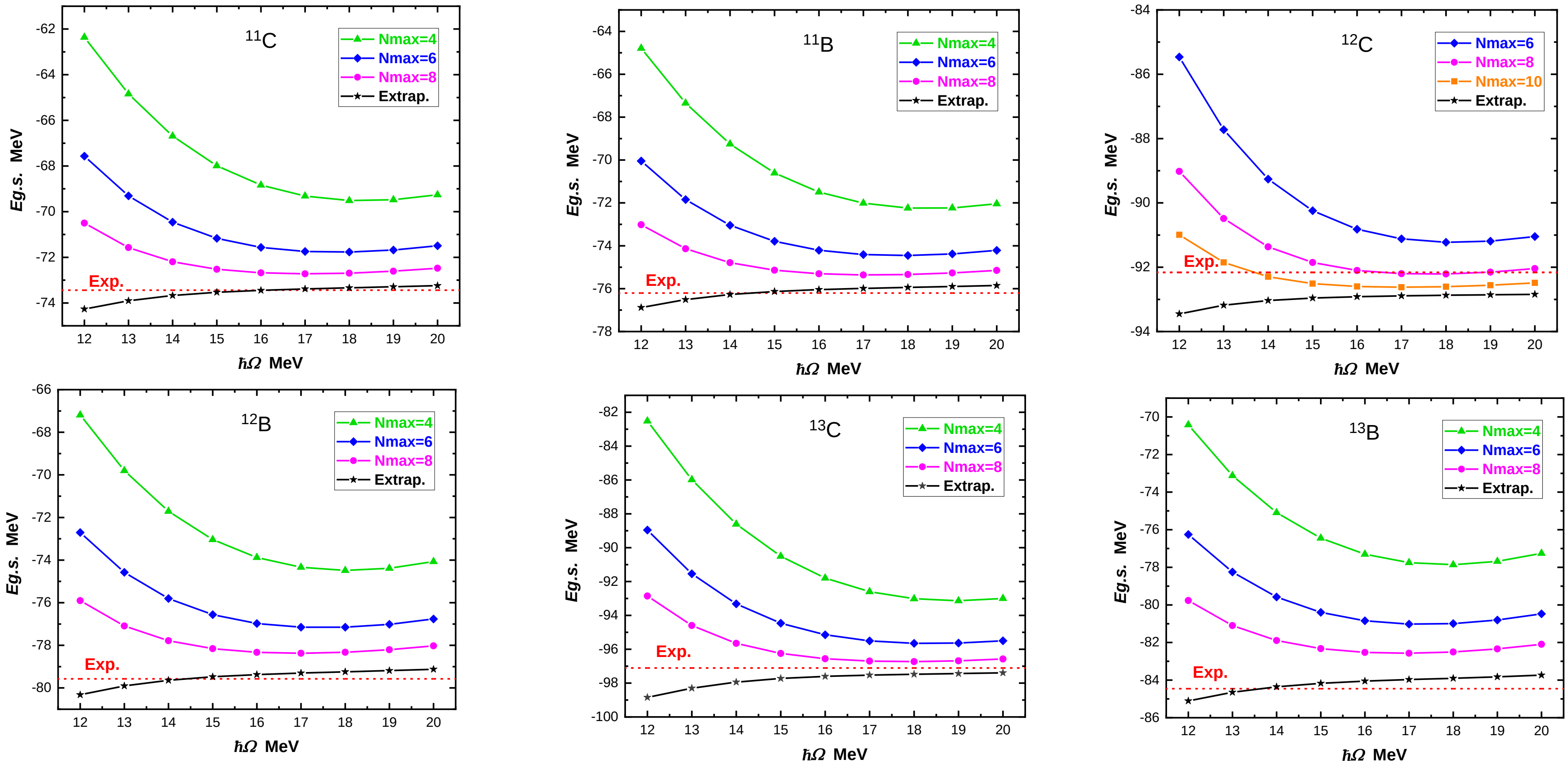
OMP_Threads=4, number of nodes up to 790



Number of nodes =790, OMP_Threads up to 8

Current progress

❖ Ground state energy from Daejeon16



NCCI+Daejeon16 has good performance on the calculations of binding energies

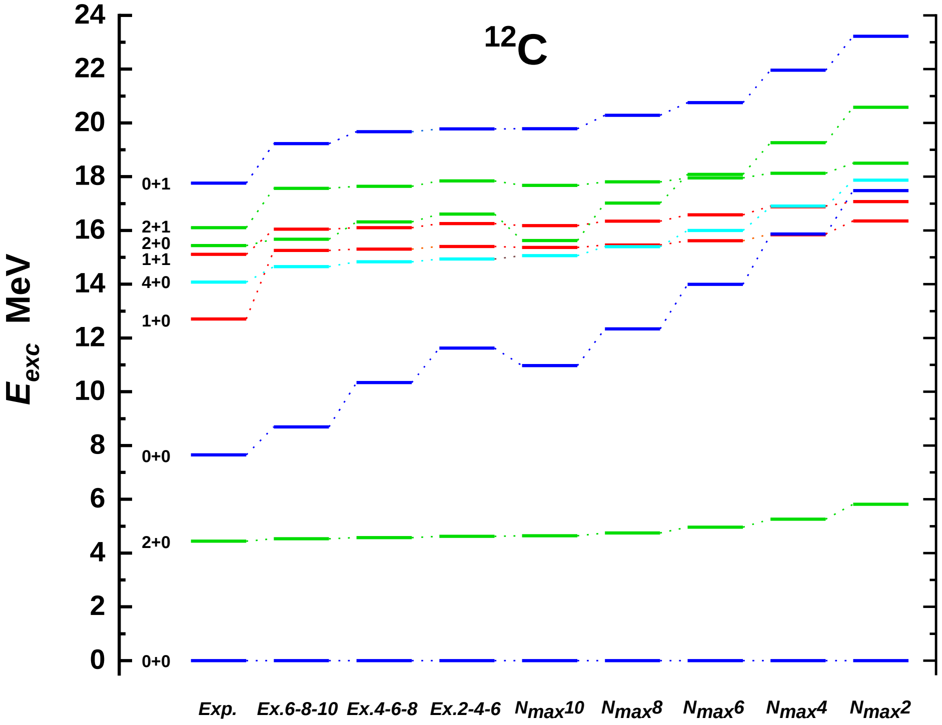
We use a simple 3-point exponential extrapolation from the reference paper

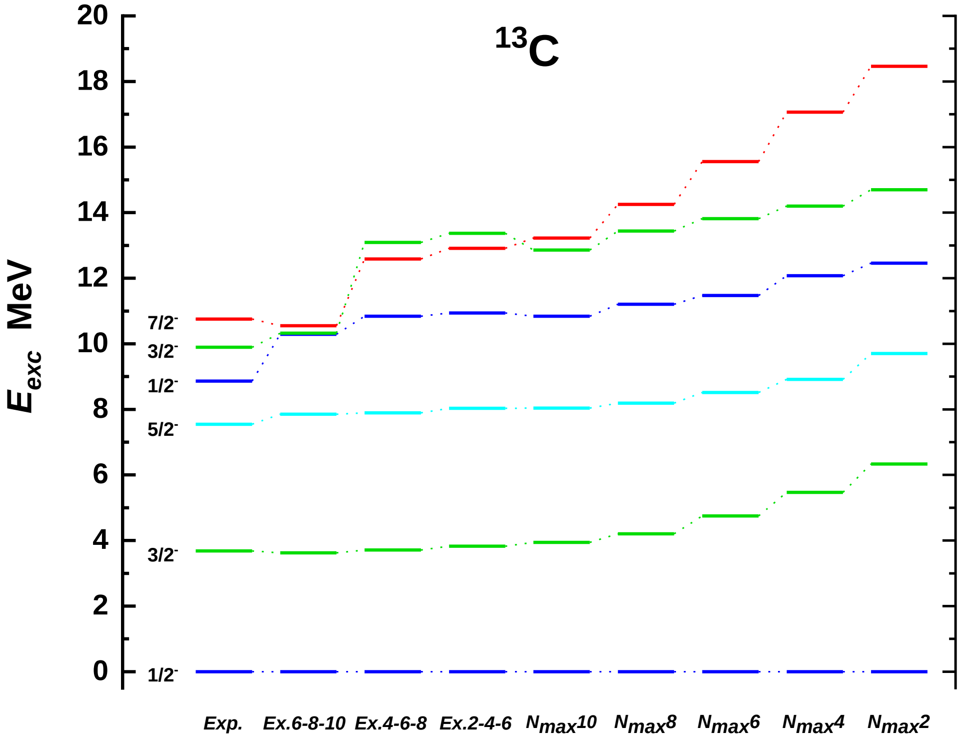
$$E^{\hbar\omega}(N_{\max}) = E^{\hbar\omega}_{\infty} + a\,\mathrm{e}^{(-b\,N_{\max})}$$

P. Maris et al., Phys. Rev. C (2021)

Present Progress

❖ Energy spectra from Daejeon16

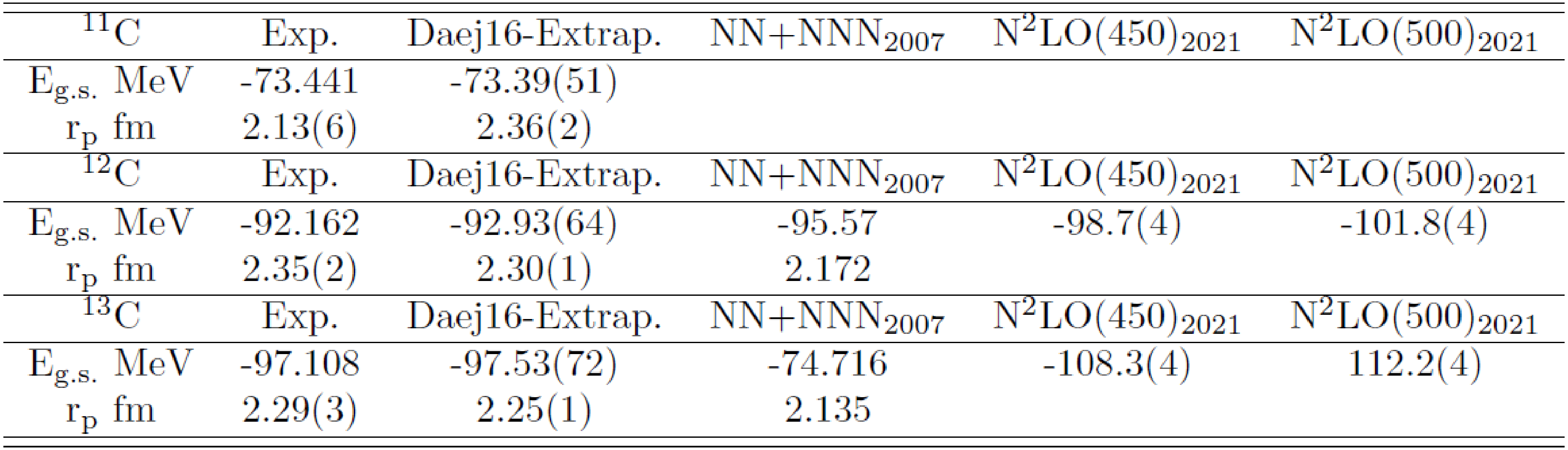




The low lying spectra of ^{12}C & ^{13}C from Daejeon16 have good agreement with experimental data

Present Progress

❖ Comparisons on ground state energies and point proton radii between Daejeon16 and other Chiral EFT interactions



^{11}C	Exp.	Daej16-Extrap.	NN+NNN$_{2007}$	N^{2}LO(450)$_{2021}$	N^{2}LO(500)$_{2021}$
$E_{g.s.}$ MeV	-73.441	-73.39(51)			
r_p fm	2.13(6)	2.36(2)			
^{12}C	Exp.	Daej16-Extrap.	NN+NNN$_{2007}$	N^{2}LO(450)$_{2021}$	N^{2}LO(500)$_{2021}$
$E_{g.s.}$ MeV	-92.162	-92.93(64)	-95.57	-98.7(4)	-101.8(4)
r_p fm	2.35(2)	2.30(1)	2.172		
^{13}C	Exp.	Daej16-Extrap.	NN+NNN$_{2007}$	N^{2}LO(450)$_{2021}$	N^{2}LO(500)$_{2021}$
$E_{g.s.}$ MeV	-97.108	-97.53(72)	-74.716	-108.3(4)	112.2(4)
r_p fm	2.29(3)	2.25(1)	2.135		

Ground state energies and point proton radii of Carbon-11, 12, 13 from Deajeon16 in good agreement with experiment, even better than NN + NNN interaction.

P. Maris et al., Phys. Rev. C (2021)

P. Navrátil et al., Phys. Rev. Lett (2007)

Present Progress

- **Comparisons on ground state energies and point proton radii between Daejeon16 and other Chiral EFT interactions**

^{12}N	Exp.	Daejeon16-Extrap.	N^{3}LO	N^{2}LO$_{opt}$
$E_{g.s.}(1^+)$ MeV	-74.041	-73.99(56)		
r_p fm	2.49(7)	2.49(2)		
^{13}N	Exp.	Daejeon16-Extrap.	LO	LO*
$E_{g.s.}(1/2^-)$ MeV	-94.105	-94.62(46)		
r_p $(1/2_1{}^+)$ fm	NA	2.47(4)	2.52	5.85

^{11}B	Exp.	Daejeon16-Extrap.	N^{2}LO(450)$_{2021}$	N^{2}LO(500)$_{2021}$
$E_{g.s.}(3/2^-)$ MeV	-76.205	-75.99(51)	-79.8(4)	-82.3(4)
r_p fm	2.21(2)	2.27(1)		
^{12}B	Exp.	Daejeon16-Extrap.	N^{2}LO(450)$_{2021}$	N^{2}LO(500)$_{2021}$
$E_{g.s.}(1^+)$ MeV	-79.575	-79.30(59)	-84.8(4)	-87.5(4)
r_p fm	2.31(7)	2.27(1)		
^{13}B	Exp.	Daejeon16-Extrap.	N^{2}LO(450)$_{2021}$	N^{2}LO(500)$_{2021}$
$E_{g.s.}(3/2^-)$ MeV	-84.454	-83.97(69)	-92.8(5)	-95.4(5)
r_p fm	2.48(3)	2.28(1)		

Ground state energies and point proton radii of Boron 11, 12, 13 & Nitrogen 12, 13 from Daejeon16 in good agreement with experiment, even better than chiral N3LO interaction.

P. Maris et al., Phys. Rev. C (2021)
M.Mosavi et al., New Astronomy (2018)

Present Progress

- **Comparisons on M1 and E2 transitions between NCCI-Daejeon16 and other calculations**

The CK-POT, CK, SFO interactions were performed in shell model calculations

Model space	^{12}C (1900)	Daejeon16 Extrapolation	Chiral NN+3N $6\hbar\Omega$	Chiral NN+3N $8\hbar\Omega$	CK-POT $2\hbar\Omega$	SFO $2\hbar\Omega$
$B(M1;1^+0\rightarrow0^+0)$	0.0145(21)	0.074(1)	0.006	0.0078	0.0048	0.0044
$B(M1;1^+1\rightarrow0^+0)$	0.951(20)	0.712(5)	0.913	1.109	0.771	0.838

Model Space	^{13}C	Daejeon16 $6\hbar\Omega$	Chiral NN+3N $6\hbar\Omega$	Chiral NN $6\hbar\Omega$	CK $2\hbar\Omega$	SFO $2\hbar\Omega$
$B(M1;3/2^-:3.68\ \mathrm{MeV}\rightarrow1/2^-_{\mathrm{g.s.}})$	0.698(72)	0.969	0.402	1.148	1.17	0.878
$B(E2;3/2^-:3.68\ \mathrm{MeV}\rightarrow1/2^-_{\mathrm{g.s.}})$	6.4(8)	5.388	2.659	2.659		

Hw = 15 MeV in Daejeon16 and chiral interactions

Electromagnetic transitions of Carbon12 & 13 from Daejeon16 are not so close to experiment, but reasonable.

T. Suzuki et al., Phys. Rev. C (2012)
P. Navrátil et al., Phys. Rev. Lett (2007)
H. Sagawa et al., EPJ Web of Conferences (2018)

Next plan

1. **The Semi-leptonic weak nuclear operators are one body and can be expressed as a product of space-spin and isospin operators (7 basic operators)**

$$M_J^{M_J}(q\mathbf{x}),$$
$$\Delta_J^{M_J}(q\mathbf{x}) \equiv \mathbf{M}_{JJ}^{M_J}(q\mathbf{x}) \cdot \frac{1}{q}\vec{\nabla},$$
$$\Delta_J^{\prime M_J}(q\mathbf{x}) \equiv -i\left[\frac{1}{q}\vec{\nabla} \times \mathbf{M}_{JJ}^{M_J}(q\mathbf{x})\right] \cdot \frac{1}{q}\vec{\nabla} = [J]^{-1}\left[-J^{1/2}\mathbf{M}_{JJ+1}^{M_J}(q\mathbf{x}) + (J+1)^{1/2}\mathbf{M}_{JJ-1}^{M_J}(q\mathbf{x})\right] \cdot \frac{1}{q}\vec{\nabla},$$
$$\Sigma_J^{M_J}(q\mathbf{x}) \equiv \mathbf{M}_{JJ}^{M_J}(q\mathbf{x}) \cdot \vec{\boldsymbol{\sigma}},$$
$$\Sigma_J^{\prime M_J}(q\mathbf{x}) \equiv -i\left[\frac{1}{q}\vec{\nabla} \times \mathbf{M}_{JJ}^{M_J}(q\mathbf{x})\right] \cdot \vec{\boldsymbol{\sigma}} = [J]^{-1}\left[-J^{1/2}\mathbf{M}_{JJ+1}^{M_J}(q\mathbf{x}) + (J+1)^{1/2}\mathbf{M}_{JJ-1}^{M_J}(q\mathbf{x})\right] \cdot \vec{\boldsymbol{\sigma}},$$
$$\Sigma_J^{\prime\prime M_J}(q\mathbf{x}) \equiv \left[\frac{1}{q}\vec{\nabla} M_J^{M_J}(q\mathbf{x})\right] \cdot \vec{\boldsymbol{\sigma}} = [J]^{-1}\left[(J+1)^{1/2}\mathbf{M}_{JJ+1}^{M_J}(q\mathbf{x}) + J^{1/2}\mathbf{M}_{JJ-1}^{M_J}(q\mathbf{x})\right] \cdot \vec{\boldsymbol{\sigma}},$$
$$\Omega_J^{M_J}(q\mathbf{x}) \equiv M_J^{M_J}(q\mathbf{x})\vec{\boldsymbol{\sigma}} \cdot \frac{1}{q}\vec{\nabla},$$

2. **Write the operators in the basis used in NCCI**

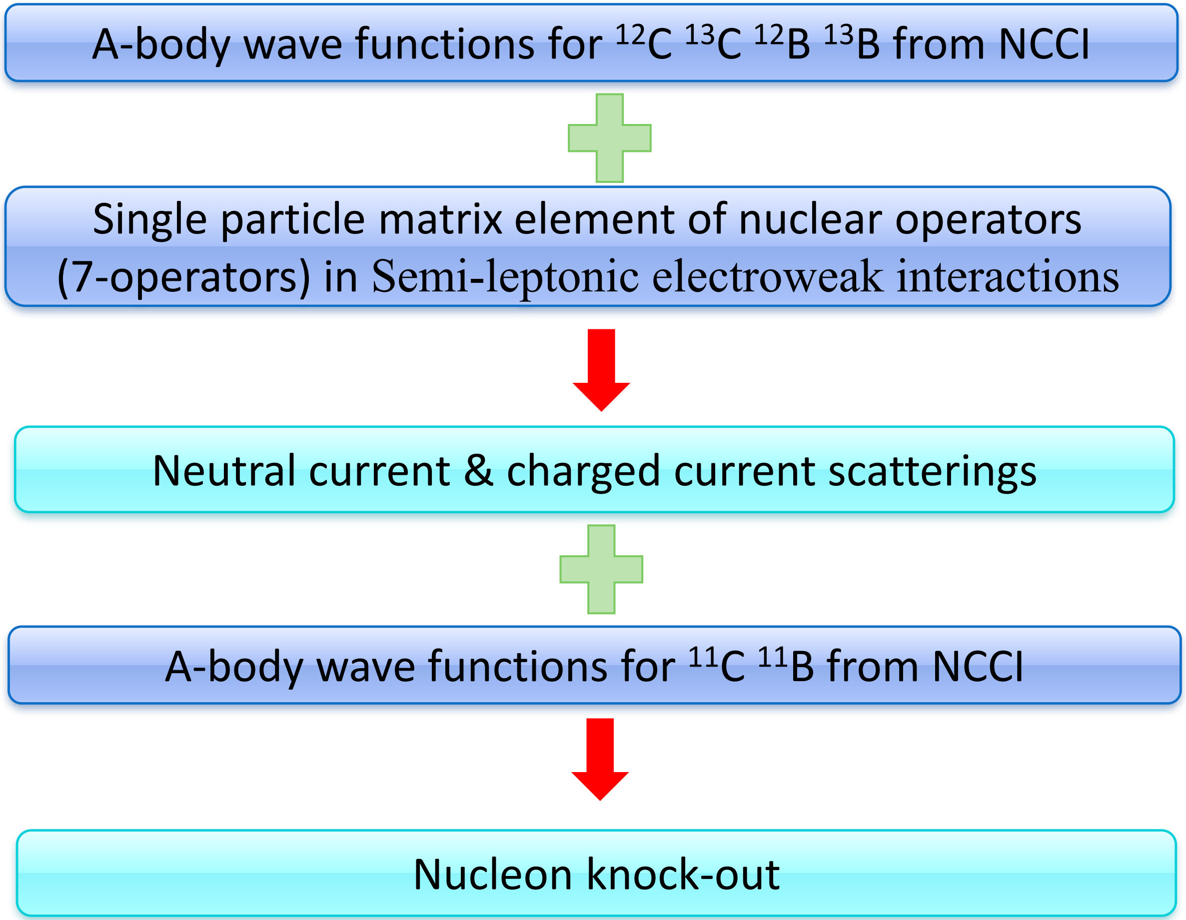
3. **Convolute with A-body wave functions from NCCI and obtain scattering matrix element**

4. **Calculate scattering cross sections**

Haxton et al., Comput. Phys. Commun(2008)

Next plan

- **From A-body wave functions to cross sections**



A-body wave functions for ^{12}C ^{13}C ^{12}B ^{13}B from NCCI

+

Single particle matrix element of nuclear operators (7-operators) in Semi-leptonic electroweak interactions

↓

Neutral current & charged current scatterings

+

A-body wave functions for ^{11}C ^{11}B from NCCI

↓

Nucleon knock-out

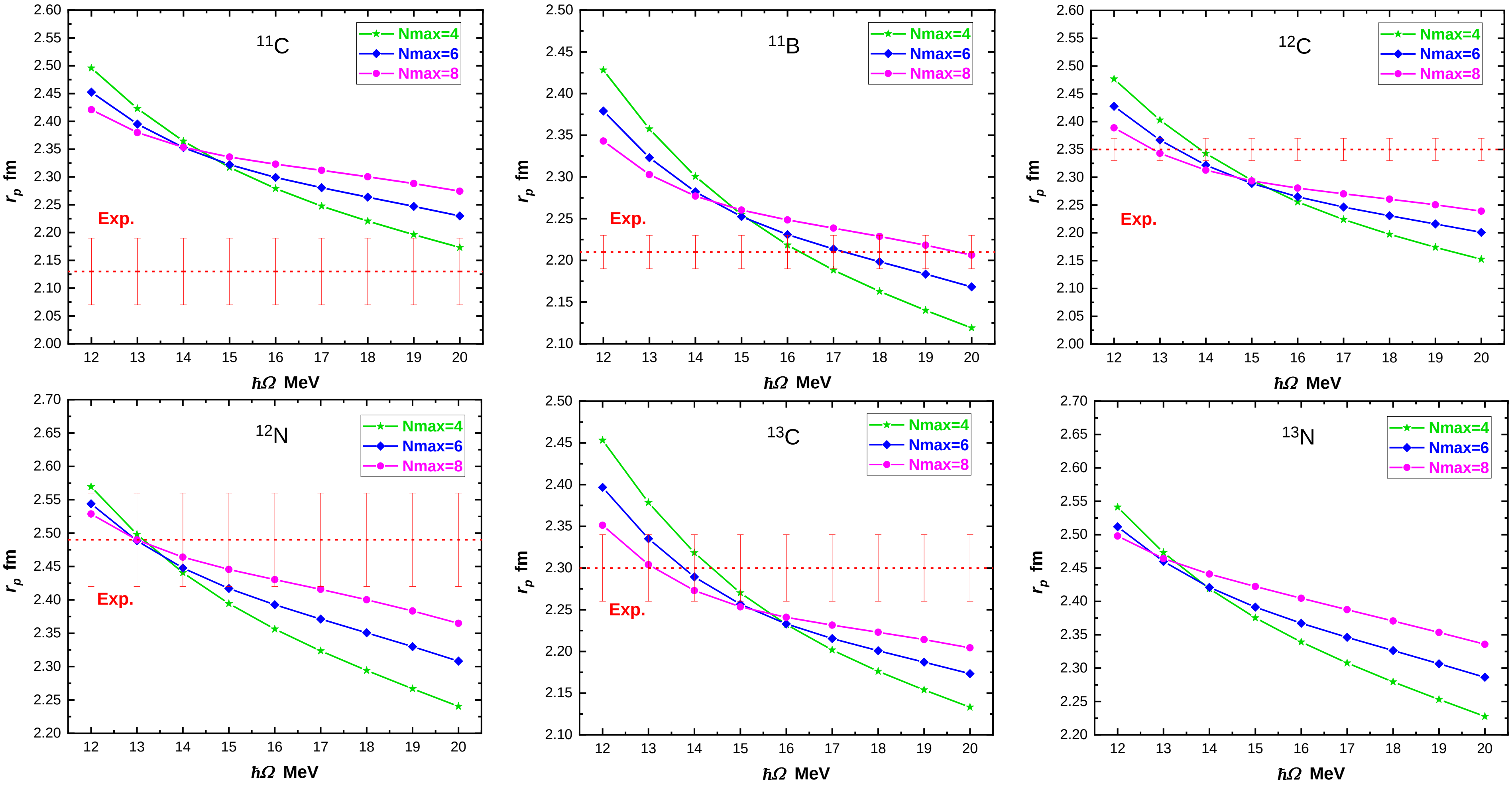
Conclusions

- Ground state energies and spectra of stable p-shell nuclei from Daejeon16 in good agreement with experiment; electromagnetic transitions are reasonable
- Increasing the basis space can significantly improve the accuracy of calculations
- Expanded the application of NCCI from electromagnetic interactions to electroweak interactions, to establish a wider connection between theoretical and experimental work
- The study of electroweak interaction can be treated as a test of Daejeon16 calculations
- Achieving world-leading scale computing on China's supercomputers

Thank you!

Present Progress

❖ Point Proton Radii of ground state from Daejeon16



We get the estimations and uncertainties of point proton radii from the "crossing points"
The point proton radii values from Deajeon16 quite close to experimental data