基于协变手征有效场论的核子-核子相互作用研究

第一届"粤港澳"核物理论坛

肖杨

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2022 年 7 月 3 日，珠海

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- Introduction
- Covariant nucleon-nucleon contact Lagrangian
- Covariant Two-pion exchange contributions
- NNLO covariant chiral nuclear force
- Summary & outlook

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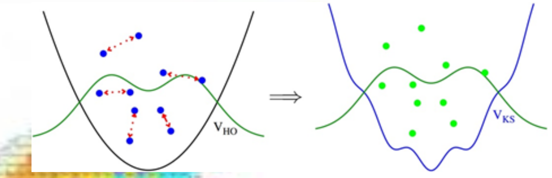
Nuclear force-basic input in nuclear physics

Theoretical laboratory of nuclear physics



DRUP DDNP 2010

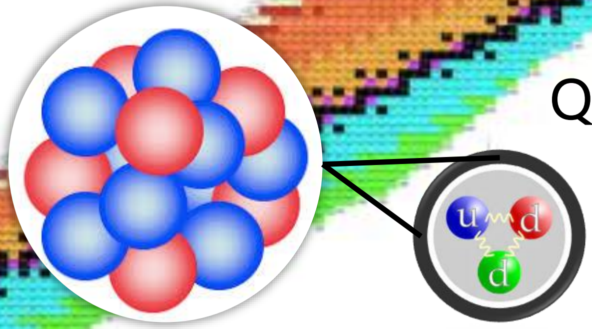
Shell model, IMSRG, …

Density Functional Theory

(Input: effective interaction)

Nucleus

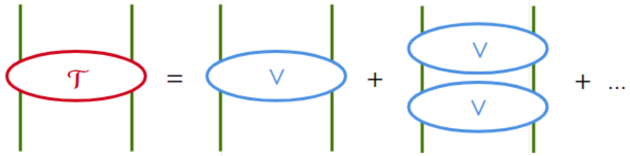
(1-10) 10^{-15}m



Quark

10^{-19}m

Quantum Field Theory

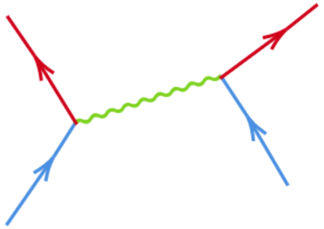


Few-body methods

$T = V + VGT$

……

$$\mathcal{L} = \bar{q}(i\gamma_\mu D^\mu - M)q - \frac{1}{4} G^{\mu\nu a} G_{\mu\nu a} + \cdots$$



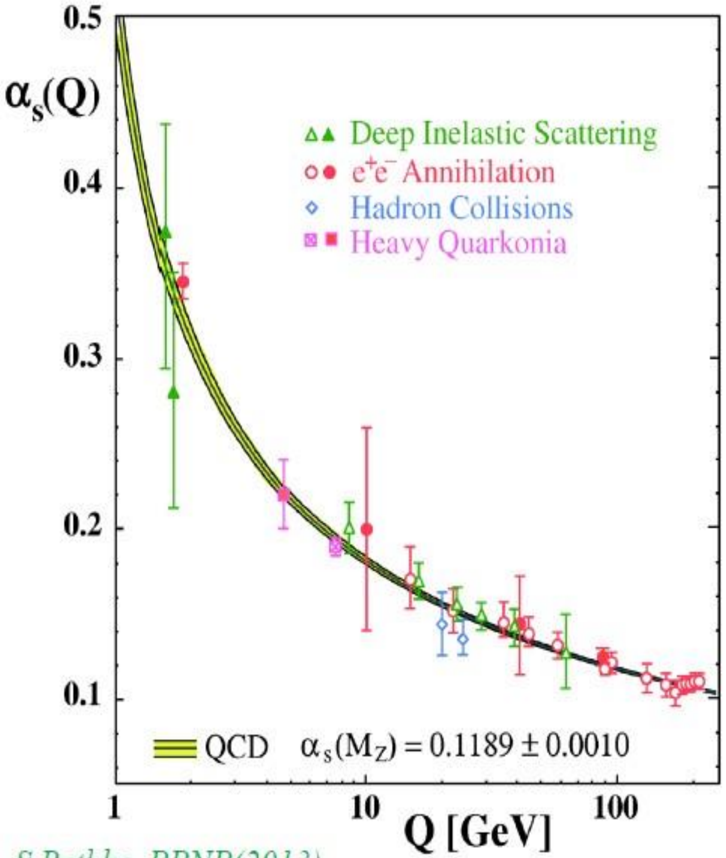
Effective Field Theory

$$\mathcal{L}_{EFT} = \psi^* \left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar}{2m} \nabla^2 \right) \psi + \frac{C_0}{2} (\psi^* \psi)^2 + \frac{D_0}{6} (\psi^* \psi)^3 + \cdots$$

Comprehensive understanding of nuclear force necessary

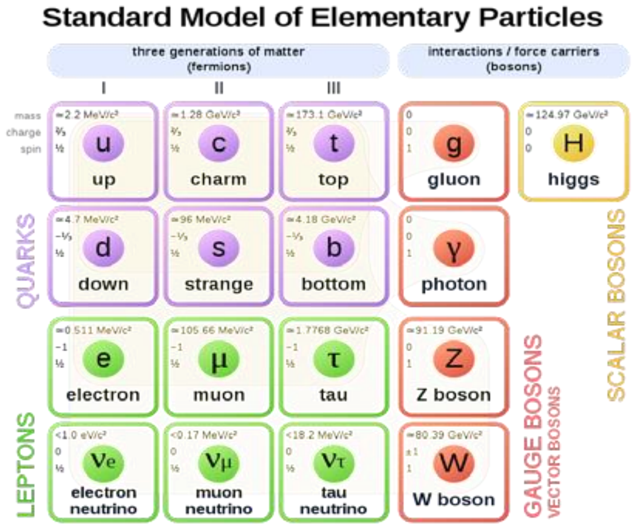
Nuclear force from QCD

$$\mathcal{L}_{QCD} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - \mathcal{M}\right)q - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha}$$

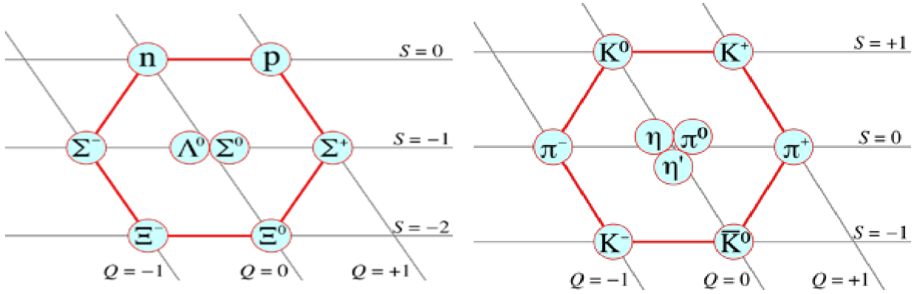


S.Bethke, PPNP(2013)

Quarks & gluons



Hadrons



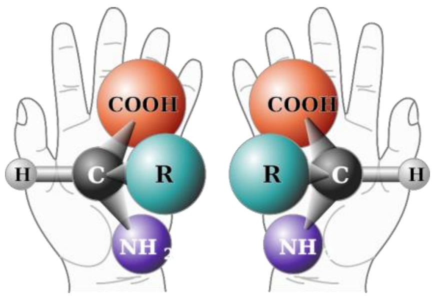
- Non-perturbative (low energy)-unsolvable
  - D.o.f.: quarks & gluons / hadrons
  - Couplings $\alpha_s > 1$

- Call for new methods (QCD based)
  1. Lattice QCD
  2. (Chiral) Effective field theory

Why chiral nuclear force (NF) ?

(Compared to phenomenological models)

- Connection to QCD - symmetries (chiral & breaking)
  - ➢ $\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{\chi EFT} \sim \sum_{\nu} \boldsymbol{c}_{\nu} \times \mathcal{L}_{\chi EFT}^{(\nu)}$
- Relevant nuclear physics degrees of freedom
  - ➢ QCD: quarks & gluons → Chiral: hadrons (non-linear realization)
- Systematic expansion parameters
  - ➢ QCD: α_s → Chiral: Q/Λ_χ $(\Lambda_\chi, m_N \sim 1\text{GeV}, Q \sim \boldsymbol{p}, q, m_\pi)$
- Error estimation



Historical overview of chiral NF

1990
Weinberg

- *PLB 251: 288 (1990)*
- *NPB 363: 3 (1991)*
- *PLB 295:114 (1992)*

1994
Ordonez, Ray and van Kolck
(N^2LO, Δ-full, local regulator)

- *PRL 72: 1982 (1994)*
- *PRC 53: 2086 (1996)*

1997
Kaiser et al.
(TPEP in DR, high partial waves)

- *NPA 625: 758 (1997)*
- *NPA 637: 395 (1998)*

2000
Epelbaum, Glockle and Meissner
(N^2LO, non-local regulator)

- *NPA 671: 295 (2000)*

2003~2005
Idaho & Julich-Bochum
(N^3LO, **high precision**)

- *PRC 68: 041001 (2003)*
- *NPA 747: 362 (2005)*

2015~2018
Idaho & Juelich-Bochum
(N^4LO)

- *PRL, 115(12): 122301 (2015)*
- *EPJA 54(5): 86 (2018)*
- *PRC 96(2): 024004 (2017)*

2018
Beijing-Chengdu-Munich (LO covariant)

- *CPC 42(1): 014103 (2018)*

2020
Idaho (N^5LO)

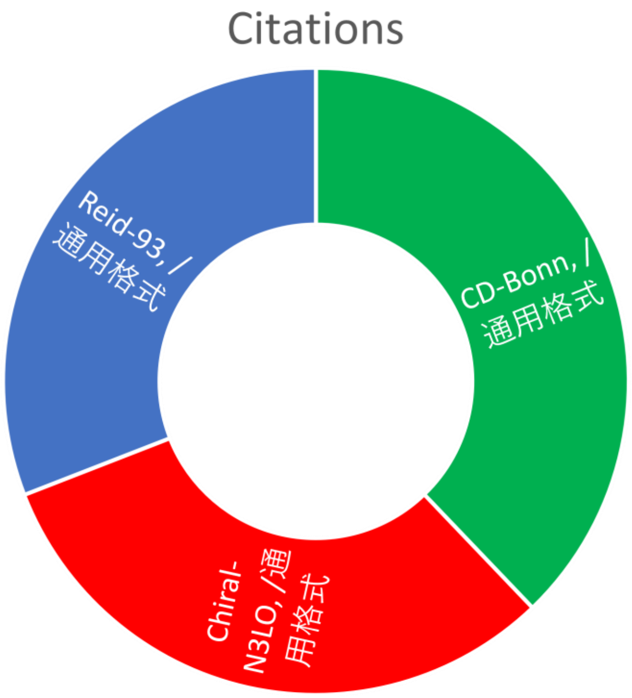
- *Front. in Phys. 8: 57 (2020)*

2022
Beijing-Munich (N^2LO covariant)

- *PRL 128 142002, (2022)*

Chiral NF vs. Phenomenological NF

D. Entem et al. PRC 96 024004 (2017)



	Phenomenological		Non-relativistic chiral			
	Reid93	CD-Bonn	LO	NLO	N^2LO	N^3LO
Parameters	50	38	2	9	9	24
χ^2/datum	1.03	1.02	94	36.7	5.28	1.27

- Chiral NF (model independent) comparable to phenomenological NF in precision

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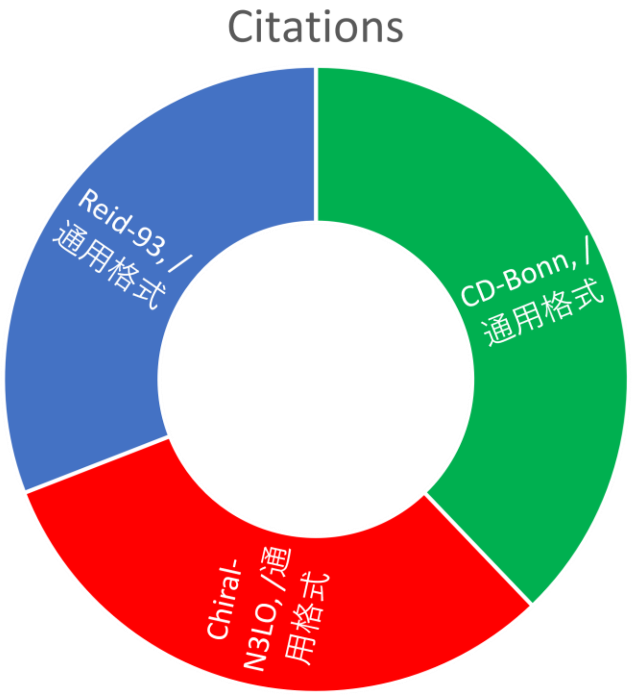

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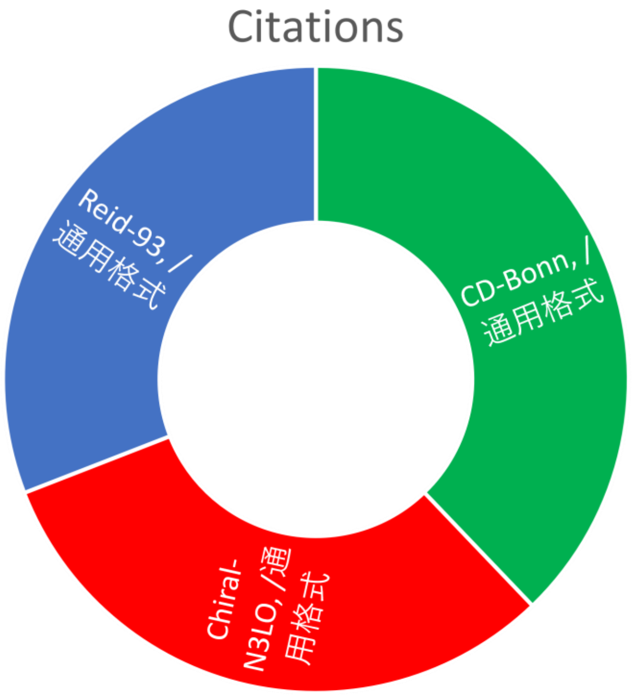

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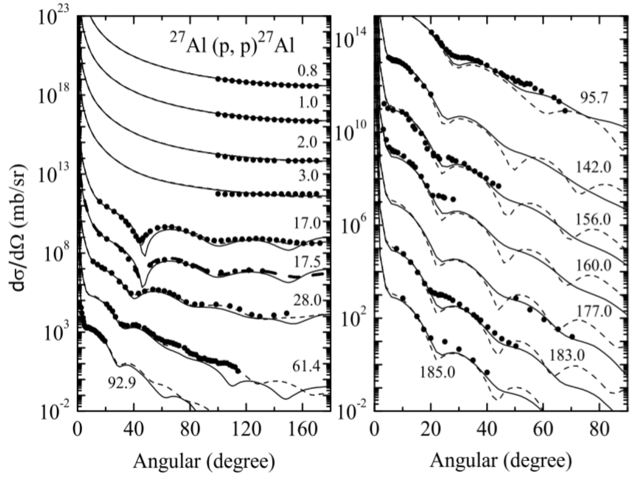
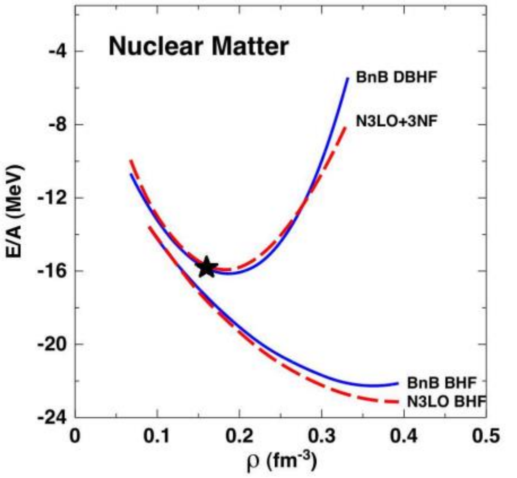
- Chiral NF (model independent) comparable to phenomenological NF in precision
- Done ? – no (yet) ➡ RG, Ay puzzle, …

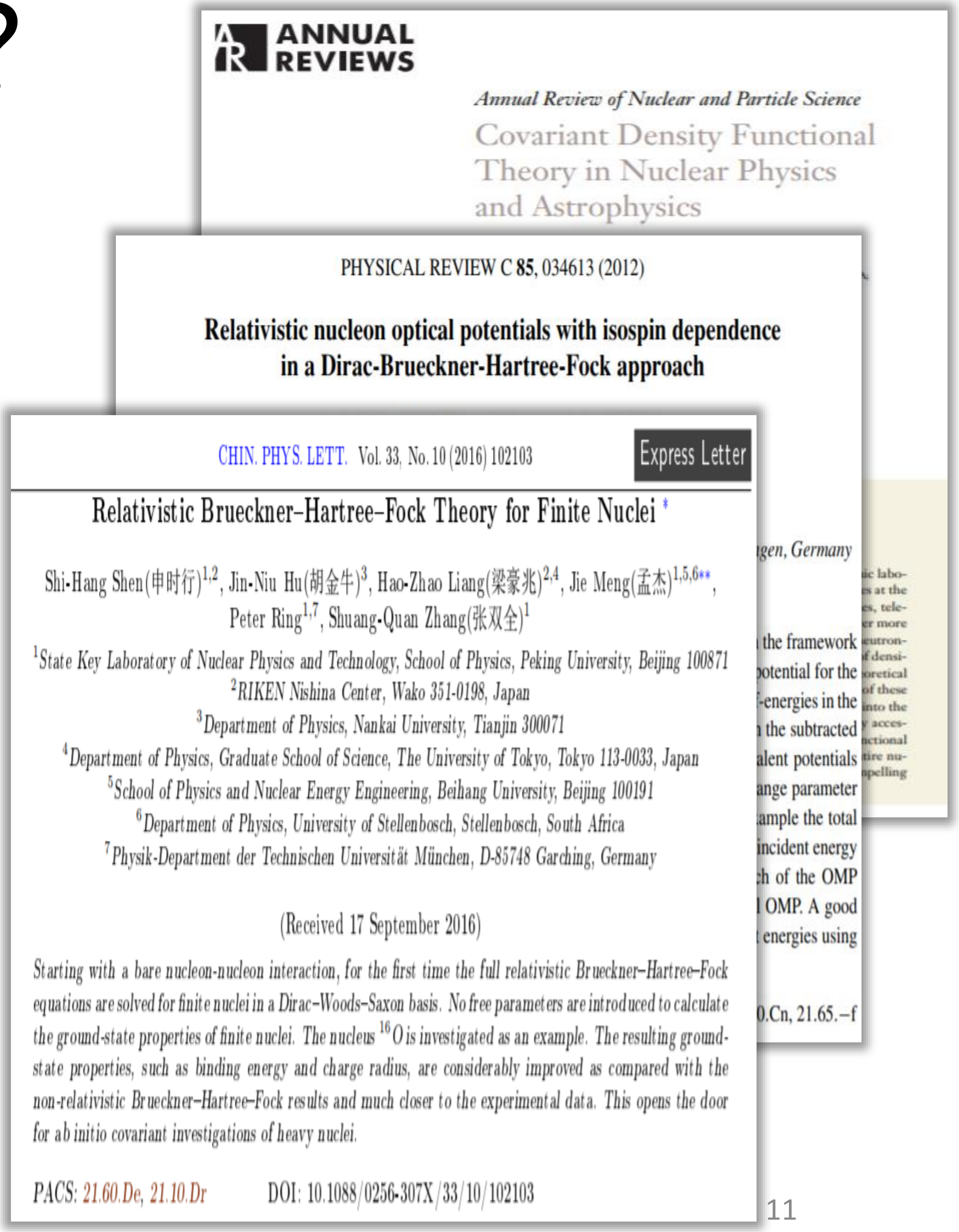
Why covariant chiral NF?





F. Sammarruca et al. PRC 86 054317 (2012)

- NR chiral NF cannot be used in covariant nuclear methods
- Bare NF input for covariant methods: Bonn potential
  - ➢ Model independent ?
  - ➢ Error estimation ?



ANNUAL REVIEWS

Annual Review of Nuclear and Particle Science

Covariant Density Functional Theory in Nuclear Physics and Astrophysics

PHYSICAL REVIEW C **85**, 034613 (2012)

Relativistic nucleon optical potentials with isospin dependence in a Dirac-Brueckner-Hartree-Fock approach

CHIN. PHYS. LETT. Vol. 33, No. 10 (2016) 102103 Express Letter

Relativistic Brueckner–Hartree–Fock Theory for Finite Nuclei *

Shi-Hang Shen(申时行)[1,2], Jin-Niu Hu(胡金牛)[3], Hao-Zhao Liang(梁豪兆)[2,4], Jie Meng(孟杰)[1,5,6**], Peter Ring[1,7], Shuang-Quan Zhang(张双全)[1]

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[2]RIKEN Nishina Center, Wako 351-0198, Japan
[3]Department of Physics, Nankai University, Tianjin 300071
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[5]School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191
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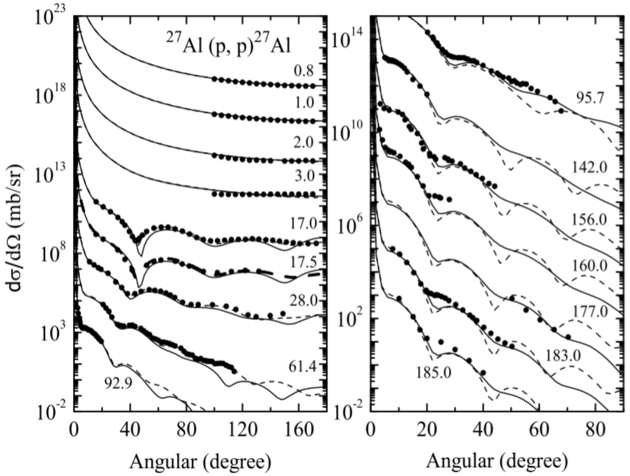
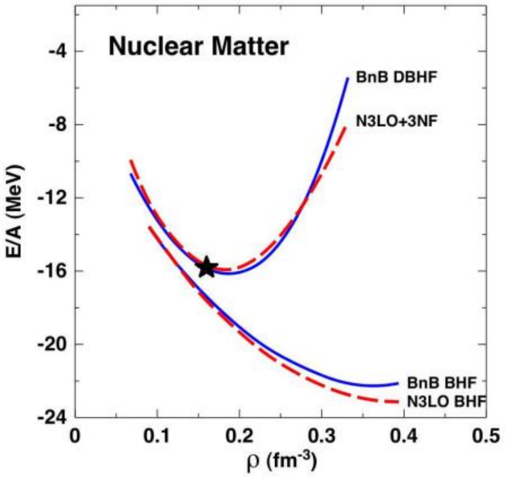
(Received 17 September 2016)

Starting with a bare nucleon-nucleon interaction, for the first time the full relativistic Brueckner–Hartree–Fock equations are solved for finite nuclei in a Dirac–Woods–Saxon basis. No free parameters are introduced to calculate the ground-state properties of finite nuclei. The nucleus ^{16}O is investigated as an example. The resulting ground-state properties, such as binding energy and charge radius, are considerably improved as compared with the non-relativistic Brueckner–Hartree–Fock results and much closer to the experimental data. This opens the door for ab initio covariant investigations of heavy nuclei.

PACS: 21.60.De, 21.10.Dr   DOI: 10.1088/0256-307X/33/10/102103

Why covariant chiral NF?





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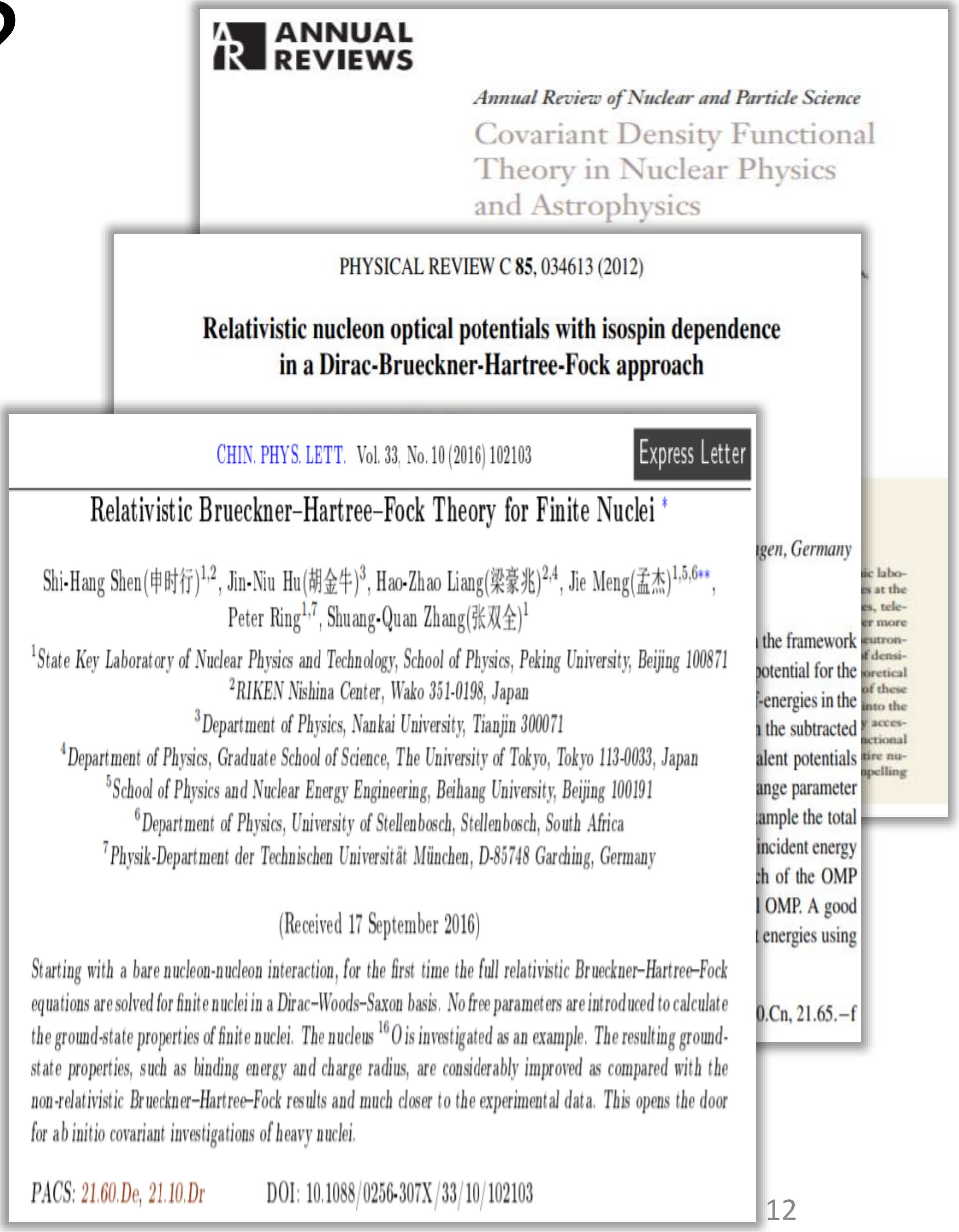
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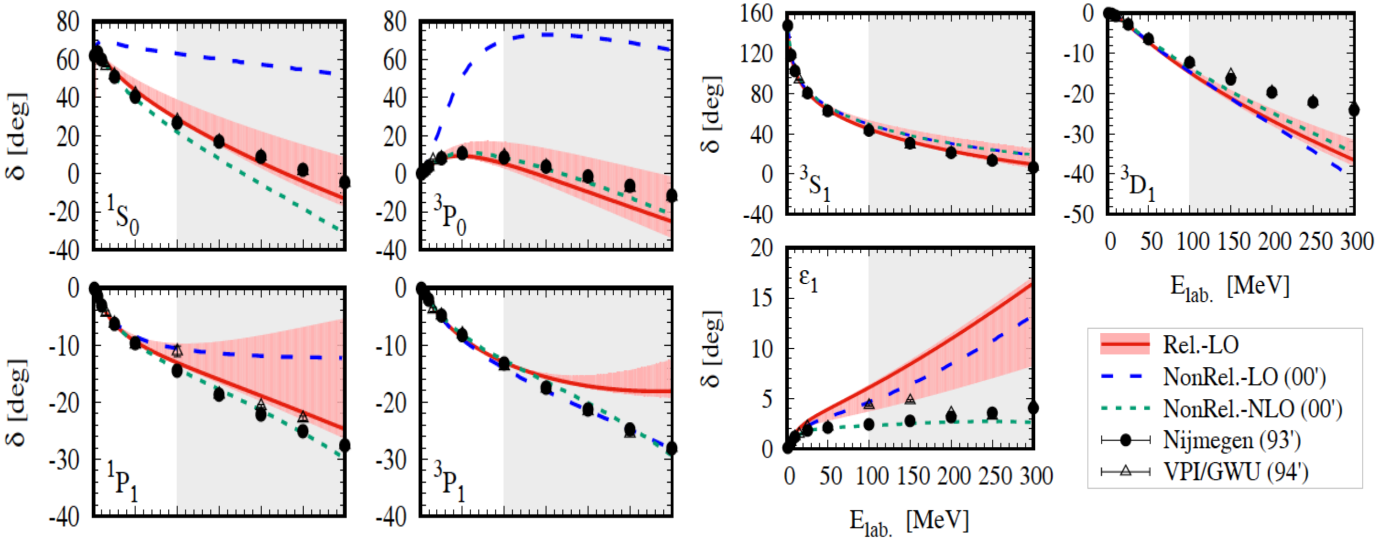
- NR chiral NF cannot be used in covariant nuclear methods
- Bare NF input for covariant methods: Bonn potential
  - ➢ Model independent ?
  - ➢ Error estimation ?

➡ Call for covariant chiral NF!

Covariant chiral NF - feasibility



Chinese Physics C Vol. 42, No. 1 (2018) 014103

Leading order relativistic chiral nucleon-nucleon interaction *

Xiu-Lei Ren(任修磊)[1,2] Kai-Wen Li(李凯文)[3] Li-Sheng Geng(耿立升)[3,4;1)]
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[5] Center for Theoretical Physics, Department of Physics, Sichuan University, Chengdu, Sichuan 610064, China
[6] Physik Department, Technische Universität München, D-85748 Garching, Germany

Abstract: Motivated by the successes of relativistic theories in studies of atomic/molecular and nuclear systems and the need for a relativistic chiral force in relativistic nuclear structure studies, we explore a new relativistic scheme to construct the nucleon-nucleon interaction in the framework of covariant chiral effective field theory. The chiral interaction is formulated up to leading order with covariant power counting and a Lorentz invariant chiral Lagrangian. We find that the relativistic scheme induces all six spin operators needed to describe the nuclear force. A detailed investigation of the partial wave potentials shows a better description of the $^1S_0$ and $^3P_0$ phase shifts than the leading order Weinberg approach, and similar to that of the next-to-leading order Weinberg approach. For the other partial waves with angular momenta $J \geqslant 1$, the relativistic results are almost the same as their leading order non-relativistic counterparts.

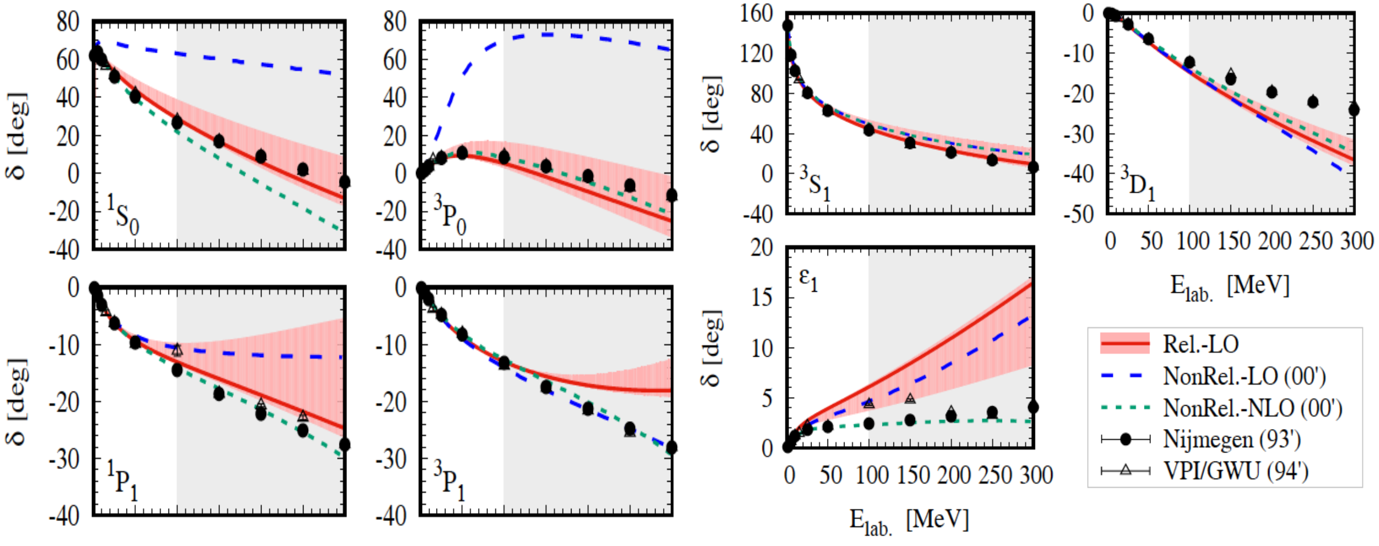
Keywords: covariant chiral perturbation theory, nucleon-nucleon interaction, relativistic scattering equation

PACS: 13.75.Cs, 21.30.-x **DOI:** 10.1088/1674-1137/42/1/014103

- LO covariant ≈ NLO non-relativistic (J=0, 1)

Good, but enough?

Covariant chiral NF - feasibility



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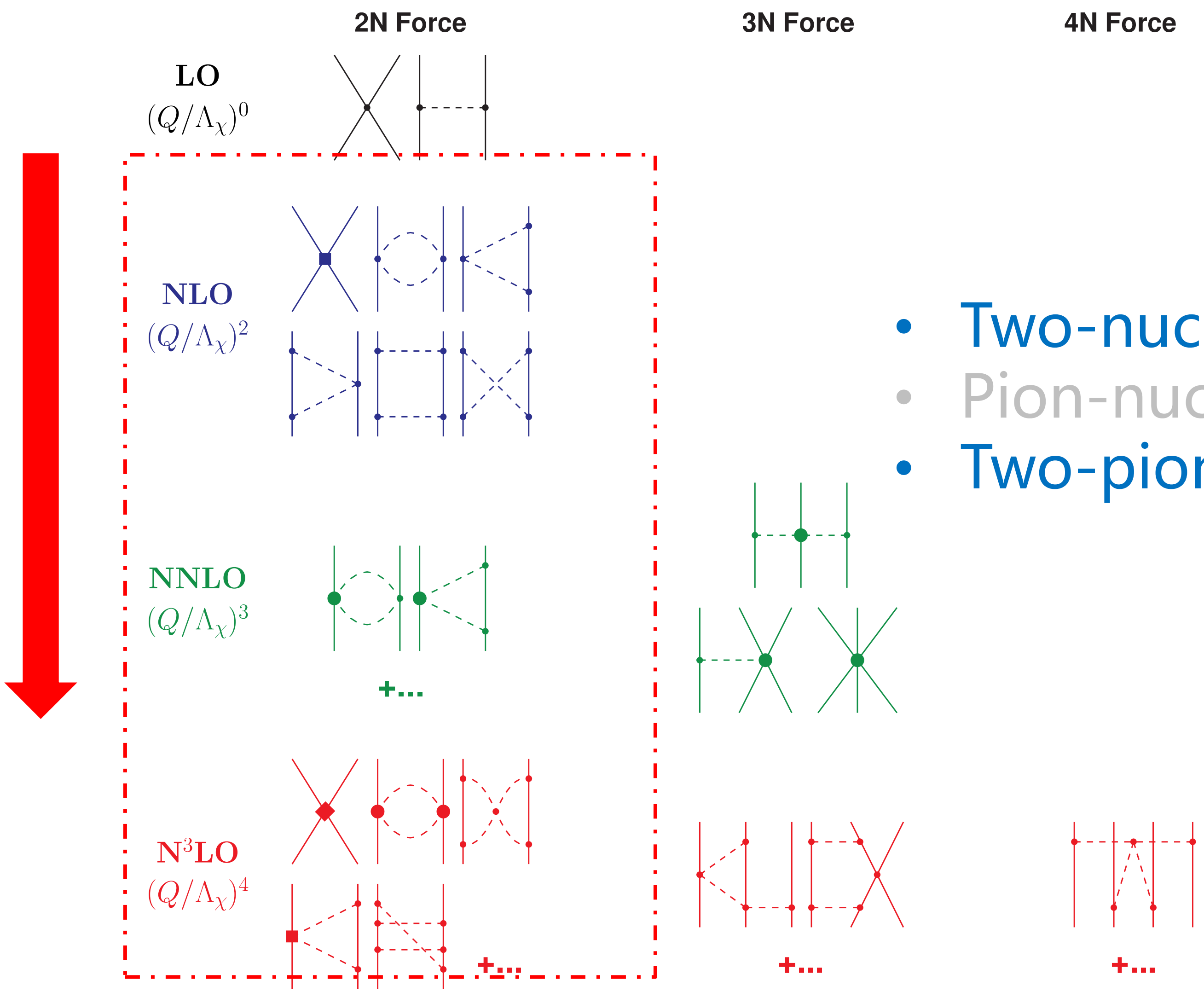
Covariant vs. non-relativistic NF

Chiral Nuclear Force Precision						
	LO	LO covariant	NLO	N^2LO	NLO/N^2LO covariant (expectation)	N^3LO
Parameters	2	4(5)	9	9	17	24
χ^2/datum	94	←here→	36.7	5.28	~ 1 ?	1.27

NLO/N^2LO covariant chiral NF on the schedule

Higher order Feynman Diagrams



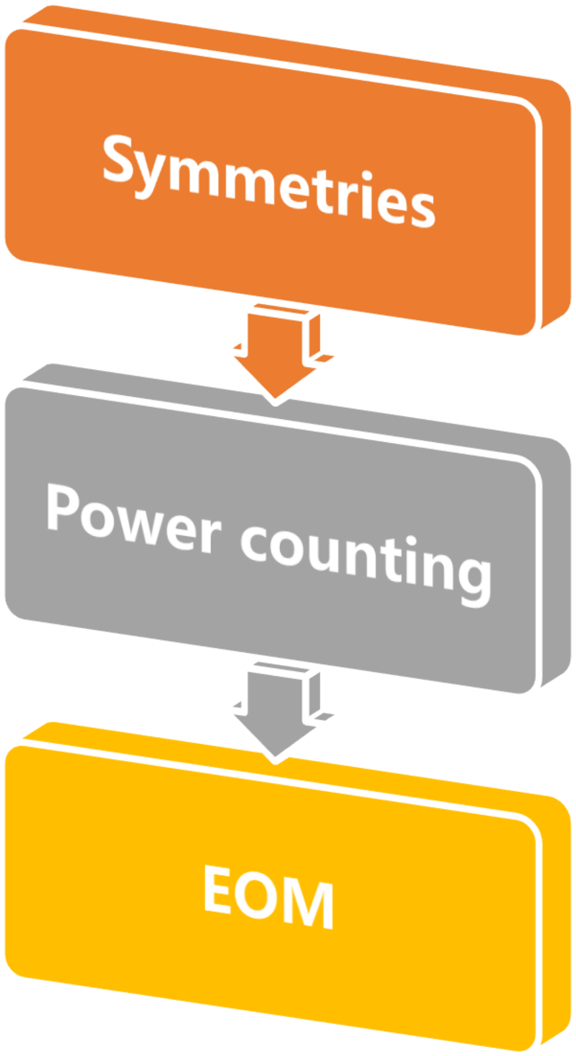
Key inputs

- Two-nucleon contact terms (short range)
- Pion-nucleon vertices
- Two-pion exchange (medium range)

CONTENTS

Covariant Lagrangian

- **Symmetries**
  - Lorentz
  - Chiral
  - Charge (C), Parity (P), Time reversal (T)
    Hermitian conjugation (H.c.)
- **Power counting**
- **Equation of motion (EOM)**
  - Remove redundant terms



Covariant Lagrangian - Symmetries

- Lorentz: α, β, γ … …
- Chiral: Matter field $\psi \to K\psi$
- Hermitian: No additional constrain
- Parity & Charge: Important !
- Time reversal: CPT theorem

Operators transform properties

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
h.c.	+	−	+	+	+	+	−	+
$\mathcal{O}$	0	1	0	0	0	−	0	1

Guide

$$\frac{1}{(2m)^{N_d}}\left(\bar{\psi} i \overleftrightarrow{\partial}^{\alpha} i \overleftrightarrow{\partial}^{\beta} \ldots \Gamma_A \psi\right) \partial^\lambda \partial^\mu \ldots \left(\bar{\psi} i \overleftrightarrow{\partial}^{\sigma} i \overleftrightarrow{\partial}^{\tau} \ldots \Gamma_B \psi\right),$$

✓ $\overleftrightarrow{\partial}^\alpha = \overrightarrow{\partial}^\alpha - \overleftarrow{\partial}^\alpha$

✓ $\partial^\alpha = \partial^\alpha(\bar{\psi}\Gamma\psi)$

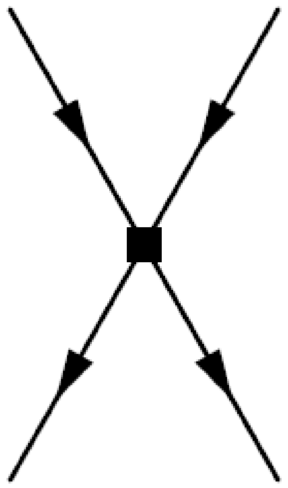
Covariant Lagrangian – Power counting

- Expressions: $\frac{1}{(2m)^{N_d}}\left(\bar{\psi} i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\ldots\Gamma_A\psi\right)\partial^{\lambda}\partial^{\mu}\ldots\left(\bar{\psi} i\overleftrightarrow{\partial}^{\sigma} i\overleftrightarrow{\partial}^{\tau}\ldots\Gamma_B\psi\right)$

N_d: 4 momentum number, $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$

- Nucleon filed: $\psi = \binom{p}{n} \sim O(p^0)$, nucleon mass: $m \sim O(p^0)$,
- Clifford Algebra: $\Gamma \in \{1, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \sim O(p^0), \gamma_5 \sim O(p^1)\}$
- Nucleon momentum: $\partial(\bar{\psi}\Gamma\psi) \sim O(p^1)$, $(\bar{\psi}\overleftrightarrow{\partial}\psi) \sim O(p^0)$
- One problem: $\tilde{O}^{(n)}_{\Gamma_A\Gamma_B} = \frac{1}{(2m)^{2n}}(\bar{\psi} i\overleftrightarrow{\partial}^{\mu_1} i\overleftrightarrow{\partial}^{\mu_2}\cdots i\overleftrightarrow{\partial}^{\mu_n}\Gamma^{\alpha}_{A}\psi)(\bar{\psi} i\overleftrightarrow{\partial}_{\mu_1} i\overleftrightarrow{\partial}_{\mu_2}\cdots i\overleftrightarrow{\partial}_{\mu_n}\Gamma_{B\alpha}\psi)$

$\rightarrow O(p^0)$



Two-nucleon contact Feynman diagram

- Solution:

-up to $O(p^2)$: $n = 0,1$;

-up to $O(p^4)$: $n = 0,1,2$.

$$\frac{[(p_1+p_3)\cdot(p_2+p_4)]^n}{(2m)^{2n}} \Leftrightarrow \left[1+\frac{(s-4m^2)-u}{4m^2}\right]^n \sim \left(O(p^0)+O(p^2)\right)^n$$

Covariant Lagrangian - EOM

- **EOM:** $\gamma^{\mu}\partial_{\mu}\psi = -im\psi + \mathcal{O}(q)$
- **Further application:** $\mathcal{L}_{\chi EFT}(\Theta^{i} = \Gamma'^{\lambda}\partial_{\lambda}^{n_i}) \approx -im\mathcal{L}_{\chi EFT}(\Theta^{i} = \Gamma\partial^{n_i-1})$
- **Summary (part):**
  - $\gamma_5\gamma^{\mu} \Leftrightarrow \epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\overleftrightarrow{\partial}^{\nu}$;
  - $\sigma_{\mu\nu} \Leftrightarrow \epsilon_{\mu\nu\alpha\beta}\gamma_5\gamma^{\alpha}\overleftrightarrow{\partial}^{\beta}$;
  - $\epsilon_{\mu\nu\alpha\beta}(\bar{\psi}\overleftrightarrow{\partial}^{\mu}\overleftrightarrow{\partial}^{\nu} \ldots \Gamma\psi) = 0$;
  - ……

Γ	Γ'_{λ}	Γ''_{λ}
$\mathbb{1}$	γ_{λ}	0
γ_{μ}	$g_{\mu\lambda}1$	$-i\sigma_{\mu\lambda}$
γ_5	0	$\gamma_5\gamma_{\lambda}$
$\gamma_5\gamma_{\mu}$	$\frac{1}{2}\epsilon_{\mu\lambda\rho\tau}\sigma^{\rho\tau}$	$g_{\mu\lambda}\gamma_5$
$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\lambda\tau}\gamma_5\gamma^{\tau}$	$-i\left(g_{\mu\lambda}\gamma_{\nu} - g_{\nu\lambda}\gamma_{\mu}\right)$
$\epsilon_{\mu\nu\rho\tau}\gamma^{\tau}$	$\epsilon_{\mu\nu\rho\lambda}1$	$g_{\mu\lambda}\gamma_5\sigma_{\nu\rho} + g_{\rho\lambda}\gamma_5\sigma_{\mu\nu} + g_{\nu\lambda}\gamma_5\sigma_{\rho\mu}$
$\epsilon_{\mu\nu\rho\tau}\gamma_5\gamma^{\tau}$	$g_{\mu\lambda}\sigma_{\nu\rho} + g_{\rho\lambda}\sigma_{\mu\nu} + g_{\nu\lambda}\sigma_{\rho\mu}$	$\epsilon_{\mu\nu\rho\lambda}\gamma_5$
$\epsilon_{\mu\nu\rho\alpha}\sigma^{\alpha}_{\tau}$	$\gamma_5\gamma_{\rho}\left(g_{\lambda\nu}g_{\mu\tau} - g_{\lambda\mu}g_{\nu\tau}\right) + \gamma_5\gamma_{\nu}\left(g_{\lambda\mu}g_{\rho\tau} - g_{\lambda\rho}g_{\mu\tau}\right) + \gamma_5\gamma_{\mu}\left(g_{\lambda\rho}g_{\nu\tau} - g_{\lambda\nu}g_{\rho\tau}\right)$	$ig_{\lambda\tau}\epsilon_{\mu\nu\rho\alpha}\gamma^{\alpha} - i\epsilon_{\mu\nu\rho\lambda}\gamma_{\tau}$
$\frac{i}{2}\epsilon_{\mu\nu\rho\tau}\sigma^{\rho\tau} = \gamma_5\sigma_{\mu\nu}$	$\frac{1}{i}\left(g_{\mu\lambda}\gamma_5\gamma_{\nu} - g_{\nu\lambda}\gamma_5\gamma_{\mu}\right)$	$\epsilon_{\mu\nu\lambda\rho}\gamma^{\rho}$

N. Fettes et al. Annals Phys. 283:273 (2000)

N^3LO covariant Lagrangian

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^2\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_2$	$(\bar\psi\gamma^{\mu}\psi)(\bar\psi\gamma_{\mu}\psi)$	$\widetilde{O}_{22}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\alpha}\psi)\partial^2\partial_{\alpha}\partial^{\nu}(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^{\mu}\psi)(\bar\psi\gamma_5\gamma_{\mu}\psi)$	$\widetilde{O}_{23}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^{\beta}\partial_{\nu}\left(\bar\psi\sigma_{\alpha\beta}i\overleftrightarrow{\partial}_{\mu}\psi\right)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{24}$	$\frac{1}{16m^4}(\bar\psi\psi)\partial^4(\bar\psi\psi)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m^4}(\bar\psi\gamma^{\mu}\psi)\partial^4(\bar\psi\gamma_{\mu}\psi)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{26}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^{\mu}\psi)\partial^4(\bar\psi\gamma_5\gamma_{\mu}\psi)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{27}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}\psi)\partial^4(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{28}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\alpha}\psi)\partial_{\alpha}\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{29}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_{\alpha}i\overleftrightarrow{\partial}_{\mu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$	$\widetilde{O}_{30}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar\psi\gamma^{\mu}\psi)\partial^2\left(\bar\psi\gamma_{\mu}\psi\right)$	$\widetilde{O}_{31}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial^{\alpha}\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^{\mu}\psi)\partial^2\left(\bar\psi\gamma_5\gamma_{\mu}\psi\right)$	$\widetilde{O}_{32}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial_{\alpha}\partial^{\nu}\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{33}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\psi)\partial^2(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{38}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\nu}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_{\nu}i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{39}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^{\mu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_{\mu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{40}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{17}-\widetilde{O}_4$

O_S	$(N^{\dagger}N)(N^{\dagger}N)$	O_{11}	$(N^{\dagger}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)(N^{\dagger}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
O_T	$(N^{\dagger}\boldsymbol{\sigma}N)\cdot(N^{\dagger}\boldsymbol{\sigma}N)$	O_{12}	$i(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)(N^{\dagger}\overrightarrow{\nabla}^2N)+\text{h.c.}$
O_1	$(N^{\dagger}N)(N^{\dagger}\overrightarrow{\nabla}^2N)+\text{h.c.}$	O_{13}	$i(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)(N^{\dagger}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
O_2	$(N^{\dagger}N)(N^{\dagger}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$	O_{14}	$(N^{\dagger}\sigma^jN)(N^{\dagger}\sigma^j\overrightarrow{\nabla}^4N)+\text{h.c.}$
O_3	$i(N^{\dagger}\boldsymbol{\sigma}N)\cdot(N^{\dagger}\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)$	O_{15}	$(N^{\dagger}\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)(N^{\dagger}\sigma^j\overrightarrow{\nabla}^2N)+\text{h.c.}$
O_4	$(N^{\dagger}\sigma^jN)(N^{\dagger}\sigma^j\overrightarrow{\nabla}^2N)+\text{h.c.}$	O_{16}	$(N^{\dagger}\sigma^j\overrightarrow{\nabla}^2N)(N^{\dagger}\sigma^j\overleftarrow{\nabla}^2N)$
O_5	$(N^{\dagger}\sigma^jN)(N^{\dagger}\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$	O_{17}	$(N^{\dagger}\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)(N^{\dagger}\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
O_6	$(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)+\text{h.c.}$	O_{18}	$(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\overrightarrow{\nabla}^2N)+\text{h.c.}$
O_7	$(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^{\dagger}\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}N)$	O_{19}	$(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^{\dagger}\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}\overrightarrow{\nabla}^2N)+\text{h.c.}$
O_8	$(N^{\dagger}N)(N^{\dagger}\overrightarrow{\nabla}^4N)+\text{h.c.}$	O_{20}	$(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)+\text{h.c.}$
O_9	$(N^{\dagger}\overrightarrow{\nabla}^2N)(N^{\dagger}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)+\text{h.c.}$	O_{21}	$(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}\overleftarrow{\nabla}^2N)+\text{h.c.}$
O_{10}	$(N^{\dagger}\overrightarrow{\nabla}^2N)(N^{\dagger}\overleftarrow{\nabla}^2N)$	O_{22}	$(N^{\dagger}\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^{\dagger}\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$

Yang Xiao et al. PRC 99 024004 (2019)

Covariant: 40 vs. Non-relativistic: 24

CONTENTS

Two-pion exchange up to N^{2}LO

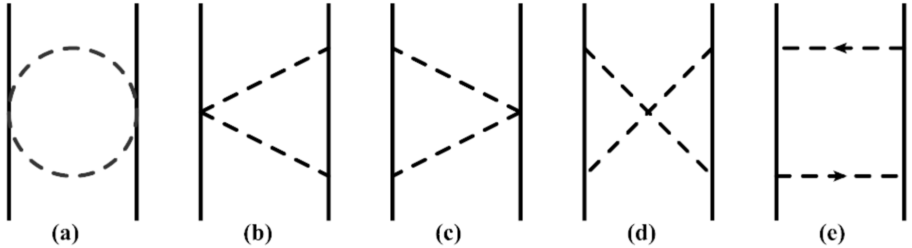
- Covariant chiral Lagrangian:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(\gamma^\mu D_\mu - m_N + \frac{g_A}{2}\gamma^\nu u_\nu \gamma_5\right)N,$$

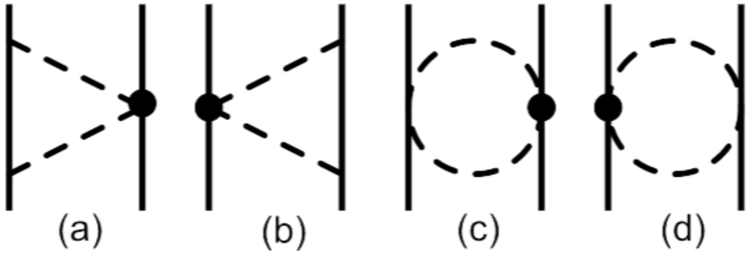
$$\mathcal{L}_{\pi N}^{(2)} = c_1\langle\chi_+\rangle\bar{N}N - \frac{c_2}{4m_N^2}\langle u^\mu u^\nu\rangle(\bar{N}D_\mu D_\nu N + \mathrm{H.c.}) + \frac{c_3}{2}\langle u^2\rangle\bar{N}N - \frac{c_4}{4}\bar{N}\gamma^\mu\gamma^\nu[u_\mu, u_\nu]N.$$

- Feynman diagrams：

$(Q/\Lambda)^2$

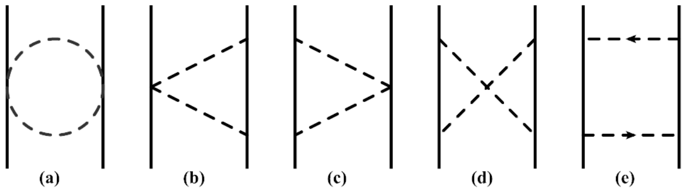


$(Q/\Lambda)^3$



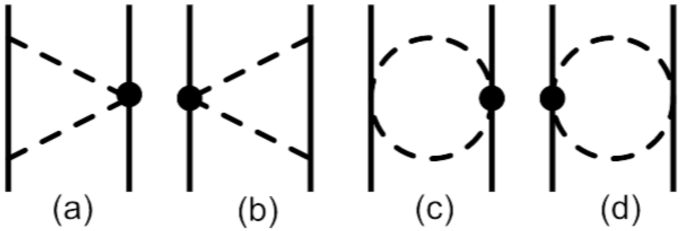
Chiral potentials

$$V_{NN}^{(2)} = \bar{u}_1 \bar{u}_2 \{$$



$$\} u_1 u_2$$

$$V_{NN}^{(3)} = \bar{u}_1 \bar{u}_2 \{$$



$$\} u_1 u_2$$

$$\bar{u}_1 \bar{u}_2 u_1 u_2 \coloneqq \bar{u}_1 u_1 \bar{u}_2 u_2 \qquad u(\boldsymbol{p}, s) = N \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E + m_N} \end{pmatrix} \chi_s, \qquad N = \sqrt{\frac{E + m_N}{m_N}}$$

T matrix & phase shifts

- On-shell T matrix: in leading order perturbation theory (for high waves)

$$T_{NN} = V_{NN}$$

- Phase shifts:

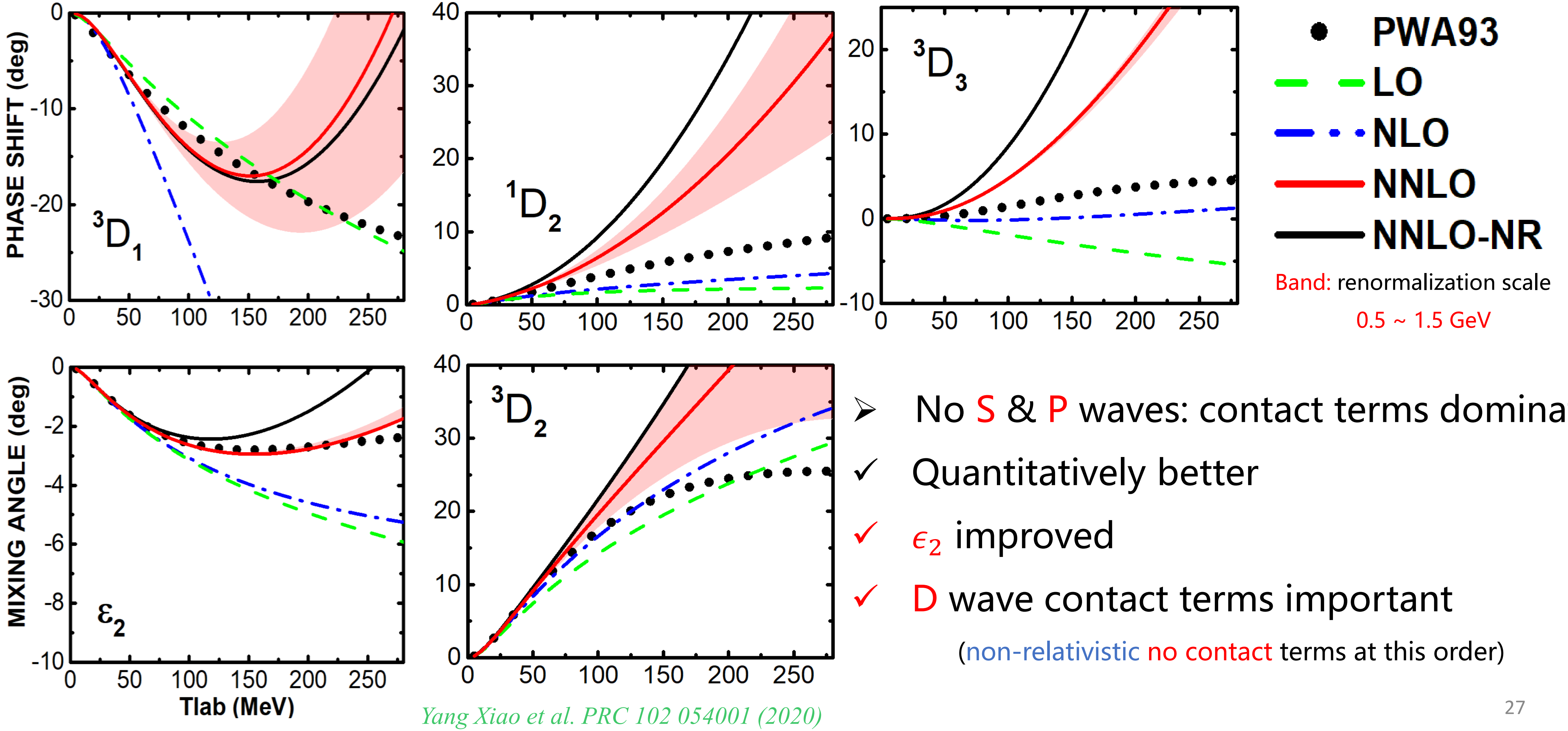
$$\delta_{LSJ} = -\frac{m_N^2 |\boldsymbol{p}|}{16\pi^2 E} \mathrm{Re}\langle LSJ | T_{NN} | LSJ \rangle,$$

$$\epsilon_J = \frac{m_N^2 |\boldsymbol{p}|}{16\pi^2 E} \mathrm{Re}\langle J-1,1,J | T_{NN} | J+1,1,J \rangle.$$

⇨ Study relativistic correction (parameters free)

(Dirac spinors vs. Pauli spinors)

D wave phase shifts

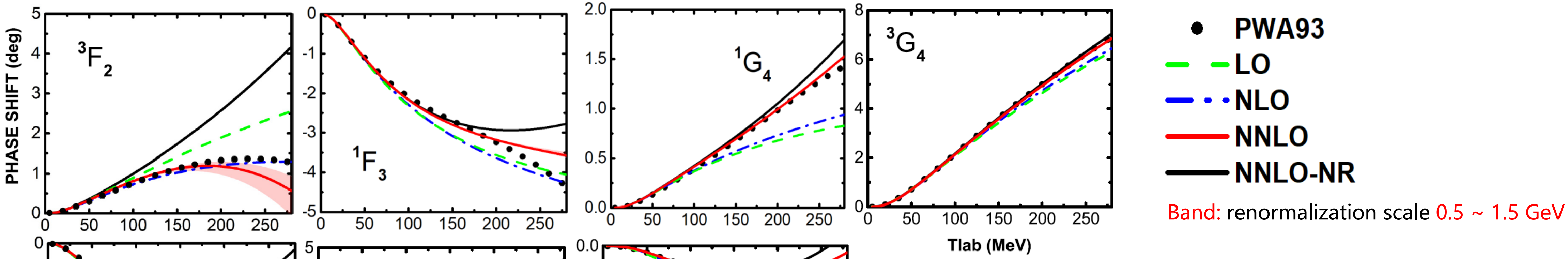


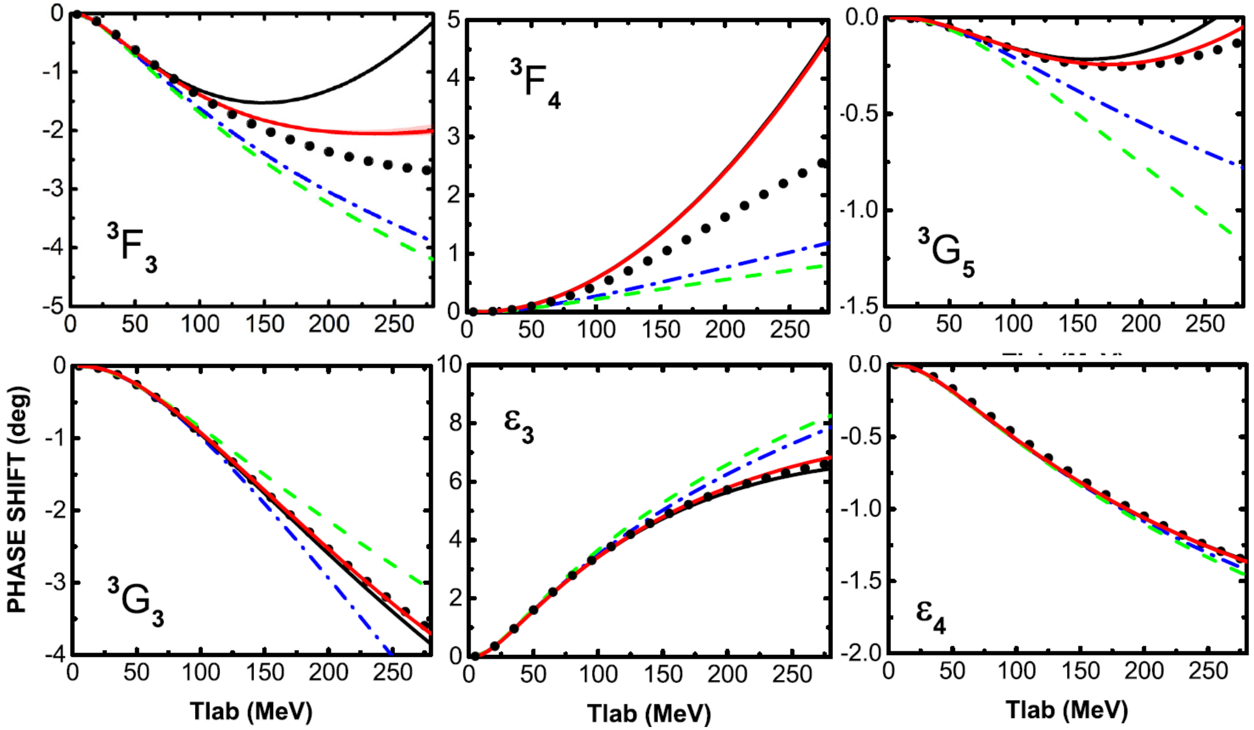
Yang Xiao et al. PRC 102 054001 (2020)

- ➢ No S & P waves: contact terms dominate
- ✓ Quantitatively better
- ✓ ϵ_2 improved
- ✓ D wave contact terms important

(non-relativistic no contact terms at this order)

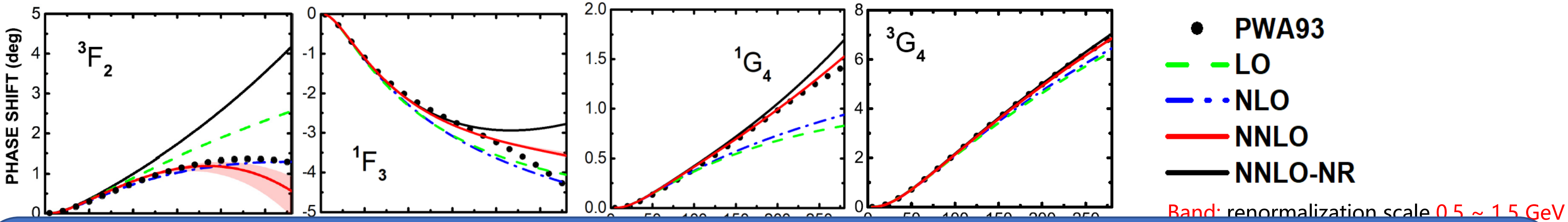
F & G wave phase shifts



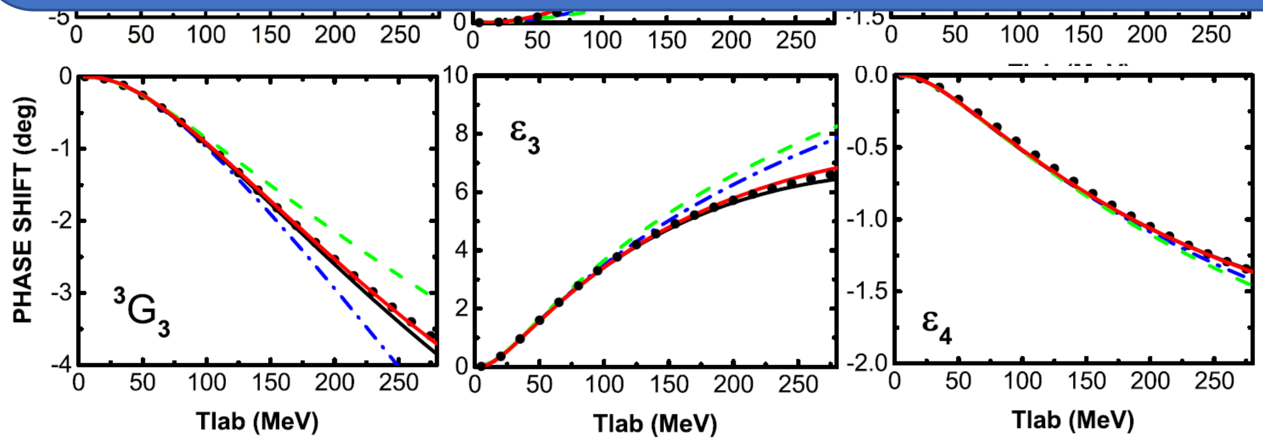


- ✓ F wave: relativistic corrections sizeable ($^3F_2$, $^1F_3$ & $^3F_3$)
- ✓ Higher partial waves: relativistic corrections insignificant

F & G wave phase shifts



Relativistic corrections improve data description for all partial waves quantitatively



✓ Higher partial waves: relativistic corrections insignificant

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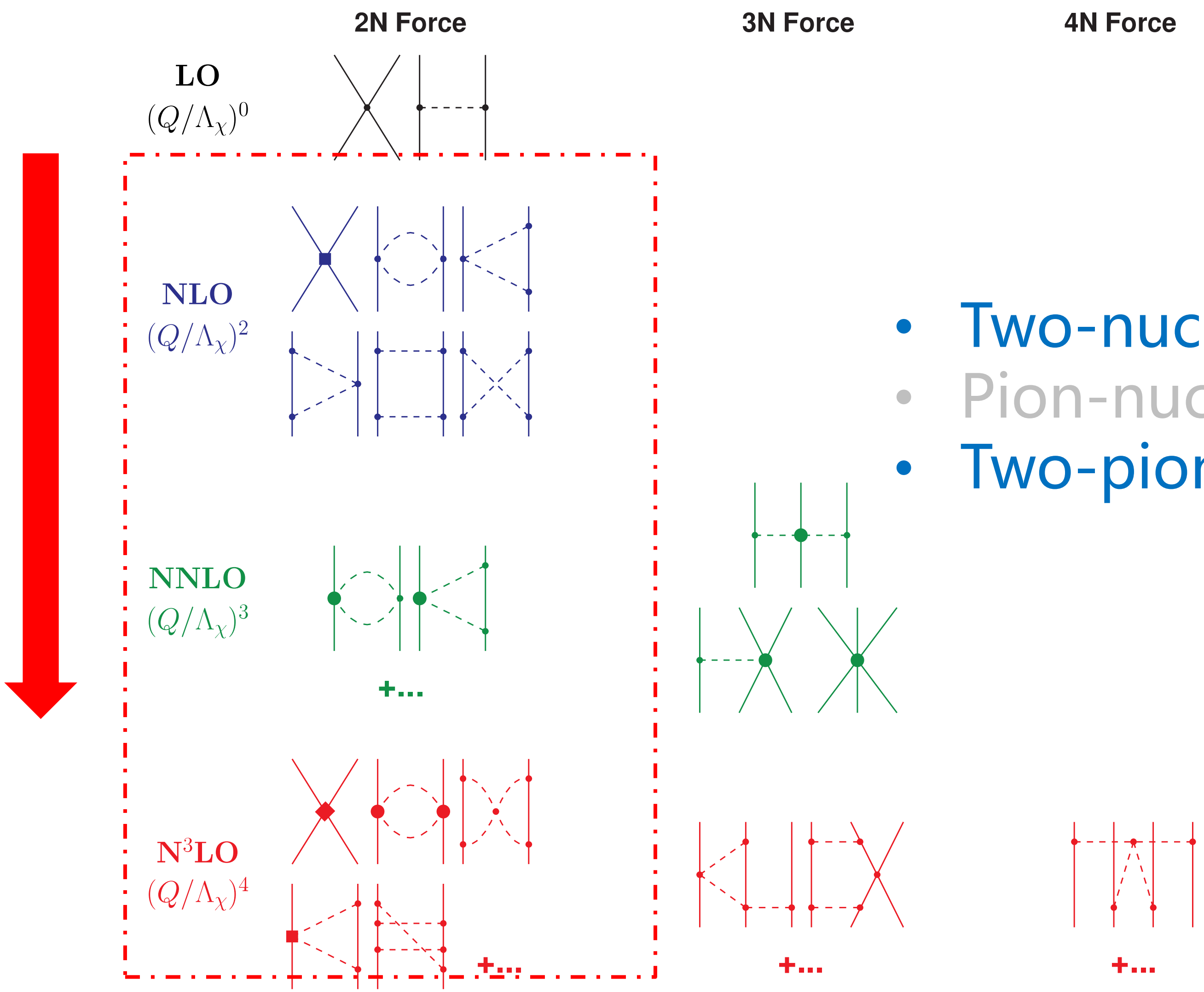
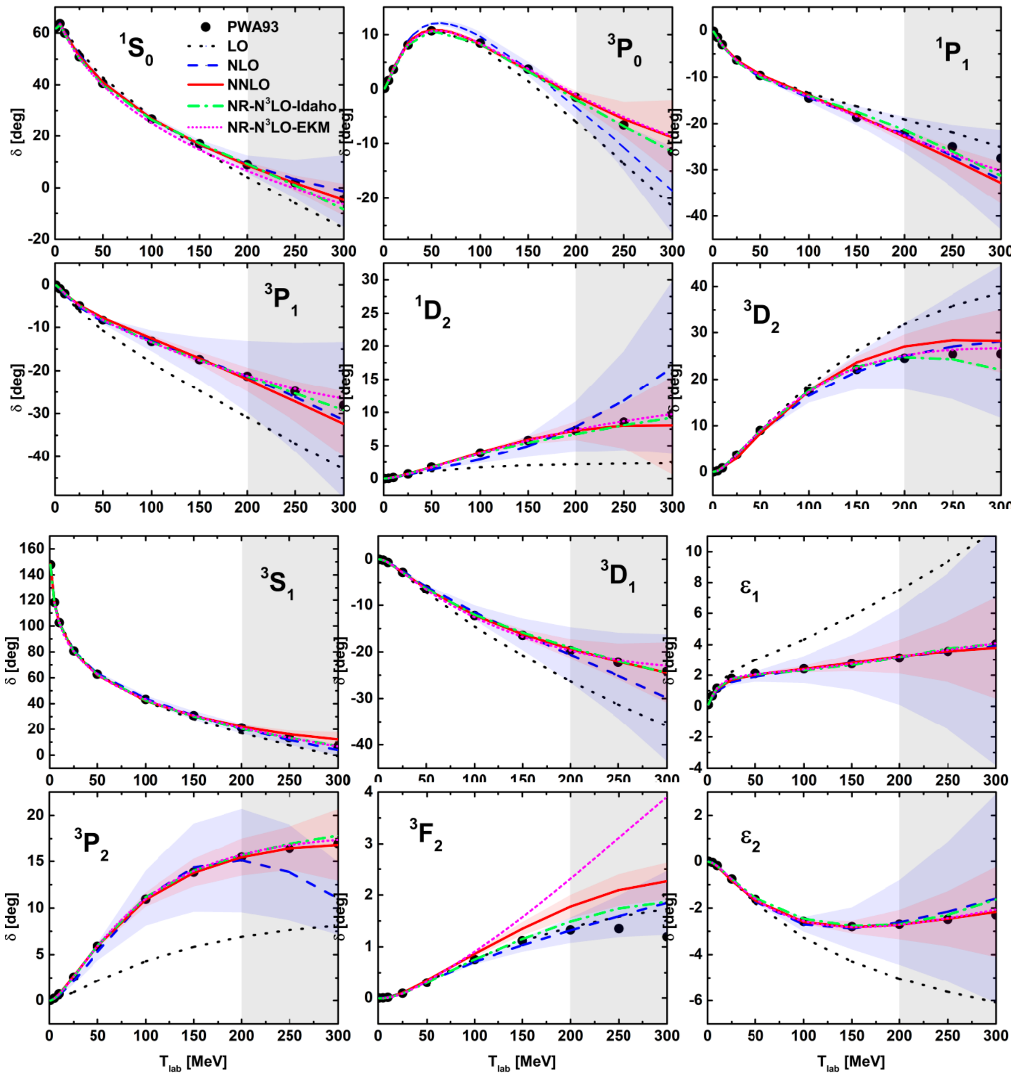
Higher order Feynman Diagrams



Key inputs

- Two-nucleon contact terms (short range)
- Pion-nucleon vertices
- Two-pion exchange (medium range)

Neutron-Proton Phase Shifts



- ✓ Characteristics
- ➢ High precision
  - **NNLO** covariant ≈ **N3LO** Heavy Baryon
- ➢ Good convergence
  - **NLO** ≈ **NNLO** (<200 MeV)
- ➢ Convincing theoretical uncertainties
  - **Bayesian method**

Jun-Xu Lu et al., PRL128 142002, (2022)

CONTENTS

Summary

1. Construct $O(q^4)$ covariant NN contact chiral Lagrangian
    - 40 terms & consistent with non-relativistic after reduction
2. Covariant two-pion exchange
    - Relativistic corrections improve data description especially for F wave
3. NNLO covariant chiral nuclear force
    - High precision, good convergence & convincing error estimation

Outlook

1. Lagrangian with isospin breaking terms for *nn* & *pp* scattering
2. TPE with Delta & Roper
3. (Covariant) RG invariance
4. Input for nuclear structure & reactions methods
5. Covariant microscopic optimal potentials

Thank you !

EFT – effective theory for underlying theory

- Main idea

  Low-energy physics independent of details of high-energy physics

- How to construct EFT

  - Identify soft / hard scales & d.o.f.s

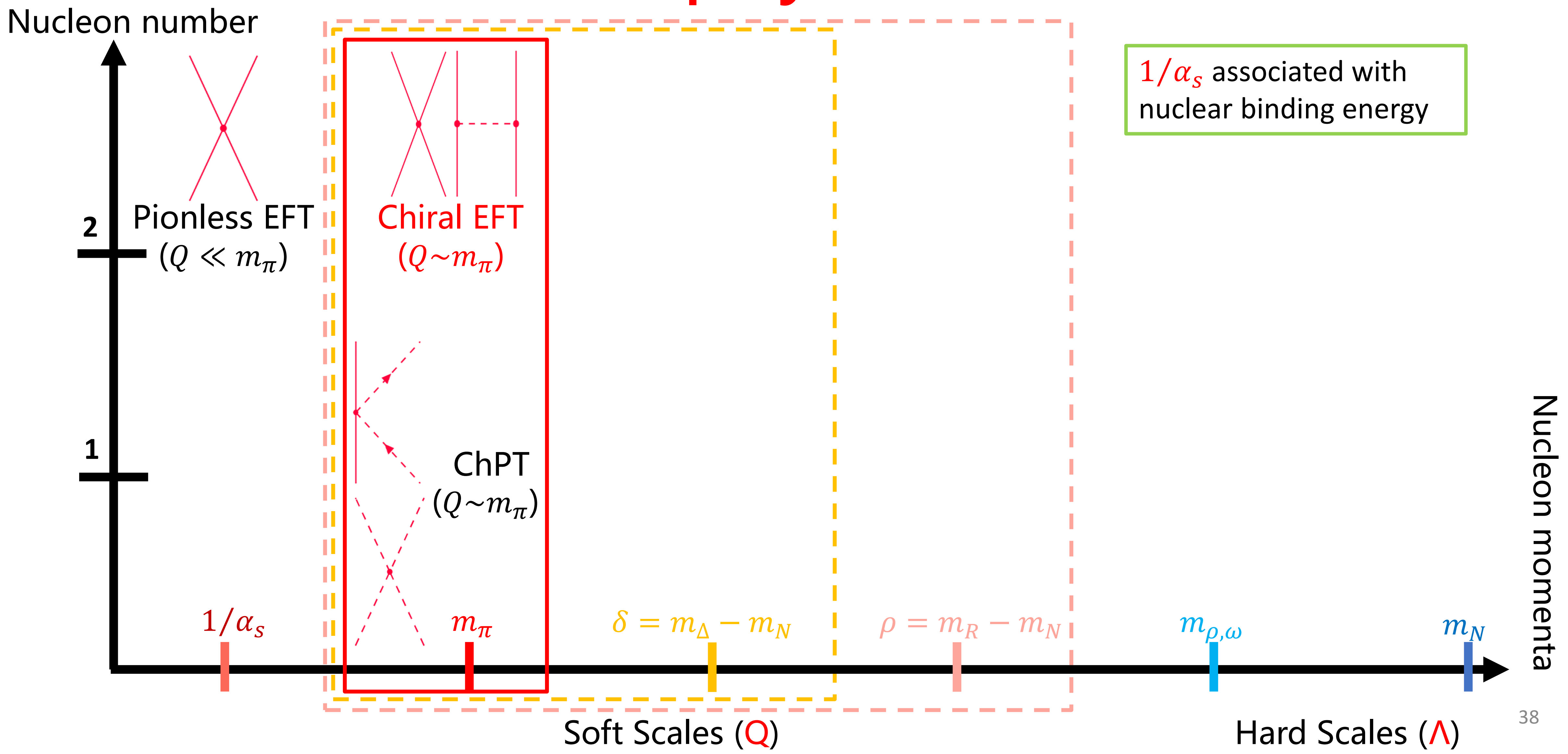
    Q (soft/low energy scale)， Λ (hard/high energy scale)

  - Construct the Lagrangian incorporating relevant symmetries

    Lorentz, chiral, …

  - Design power counting rule

EFTs for nuclear physics (few nucleons)



$1/\alpha_s$ associated with nuclear binding energy

Self consistent check

- Non-relativistic reduction: Expand nucleon field in 1/m
  - Covariant field

$$\psi(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{m}{E_p} \tilde{b}_s(\mathbf{p})\, u^{(s)}(\mathbf{p})\, e^{-ip\cdot x},$$

  - Non-relativistic field:

$$N(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} b_s(\mathbf{p})\, \chi_s\, e^{-ip\cdot x}$$

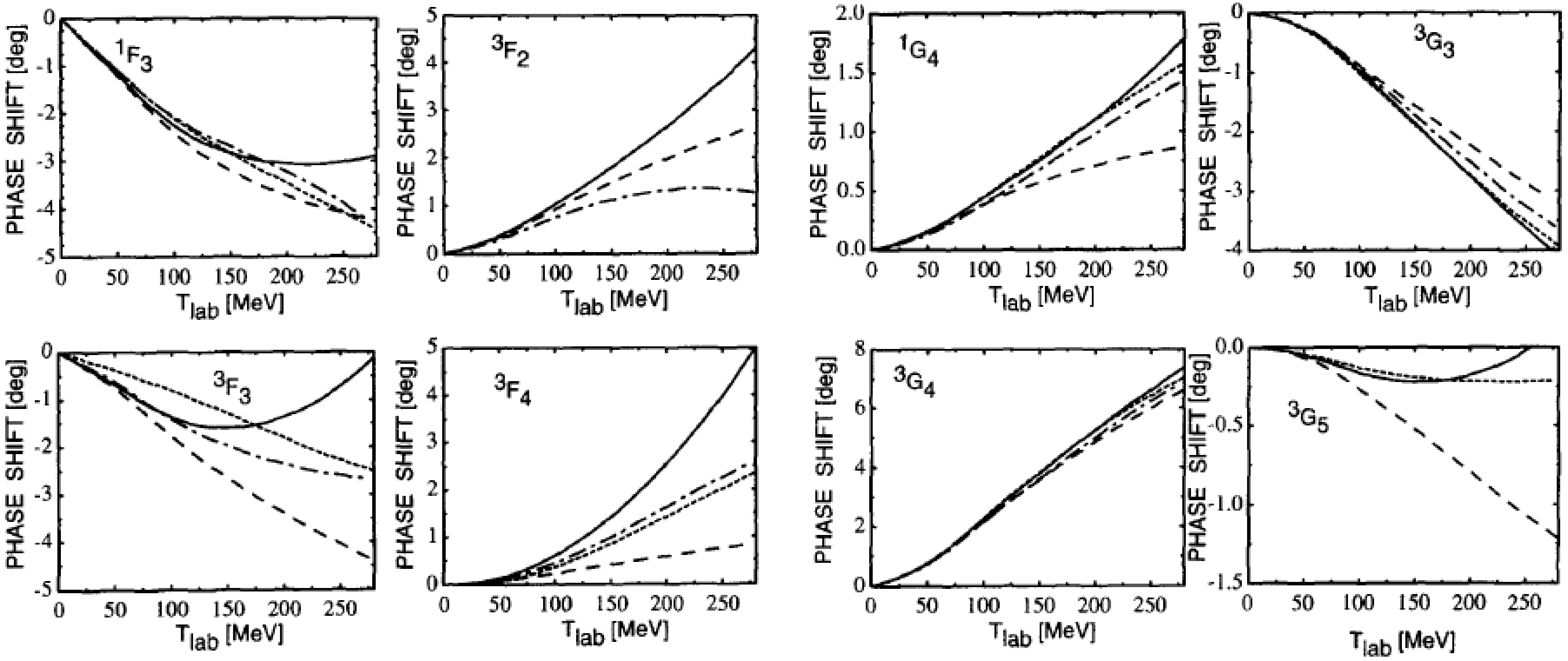
  - Expansion:

$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m}\begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} \end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix} \boldsymbol{\nabla}^2 \\ 0 \end{pmatrix} - \frac{3i}{16m^3}\begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2 \end{pmatrix} + \frac{11}{128m^4}\begin{pmatrix} \boldsymbol{\nabla}^4 \\ 0 \end{pmatrix}\right] N(x) + \mathcal{O}(Q^5)\,.$$

Covariant = non-relativistic after reduction!

Non-relativistic two-pion exchange

Dash: one-pion exchange (OPE), solid: OPE + two-pion exchange (TPE), dotted: data



N. Kaiser et al. NPA 625 758 (1997)

- 1F3 & 3F3 improved
- >= G partial waves good
- ➢ TPE important (medium range NF)

⇨ Covariant TPE ?

Complexities of covariant potentials

covariant vs. non-relativistic

Field

$$u(\boldsymbol{p},s)=\sqrt{\frac{E+m_N}{m_N}}\begin{pmatrix}1\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m_N}\end{pmatrix}\chi_s \quad \text{vs.} \quad N(s)=\chi_s$$

Propagators

$$\frac{1}{\gamma\cdot p-m_N+i\,\varepsilon} \quad \text{vs.} \quad \frac{1}{S\cdot p+i\,\varepsilon}$$

Operators

$$\gamma^\mu=(\gamma^0,\vec{\gamma}) \quad \text{vs.} \quad S^\mu=(0,\tfrac{\boldsymbol{\sigma}}{2})\text{(in rest frame)}$$

$$\gamma^0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\vec{\gamma}=\begin{pmatrix}0 & \boldsymbol{\sigma}\\ -\boldsymbol{\sigma} & 0\end{pmatrix}$$

Bilinear: (one simplest example)

$$\bar{u}_1\bar{u}_2u_1u_2=N_1^\dagger N_1N_2^\dagger N_2\left(1+\frac{E+E'}{4m_N}+\frac{EE'}{4m_N^2}\right)$$
$$-\frac{1}{4m_N^2}(N_1^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{p}'\boldsymbol{\sigma}\cdot\boldsymbol{p}N_1N_2^\dagger N_2+N_1^\dagger N_1N_2^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{p}'\boldsymbol{\sigma}\cdot\boldsymbol{p}N_2)$$
$$+\frac{N_1^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{p}'\boldsymbol{\sigma}\cdot\boldsymbol{p}N_1N_2^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{p}'\boldsymbol{\sigma}\cdot\boldsymbol{p}N_2}{4m_N^2EE'}$$

6 terms vs. 1 term

Kinematics: (TPE)

$f(p,p',m_n,m_\pi)$ vs. $g(q,m_\pi)$

Potentials = bilinear × kinematics

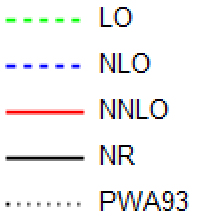
Covariant potentials much complex than non-relativistic potentials

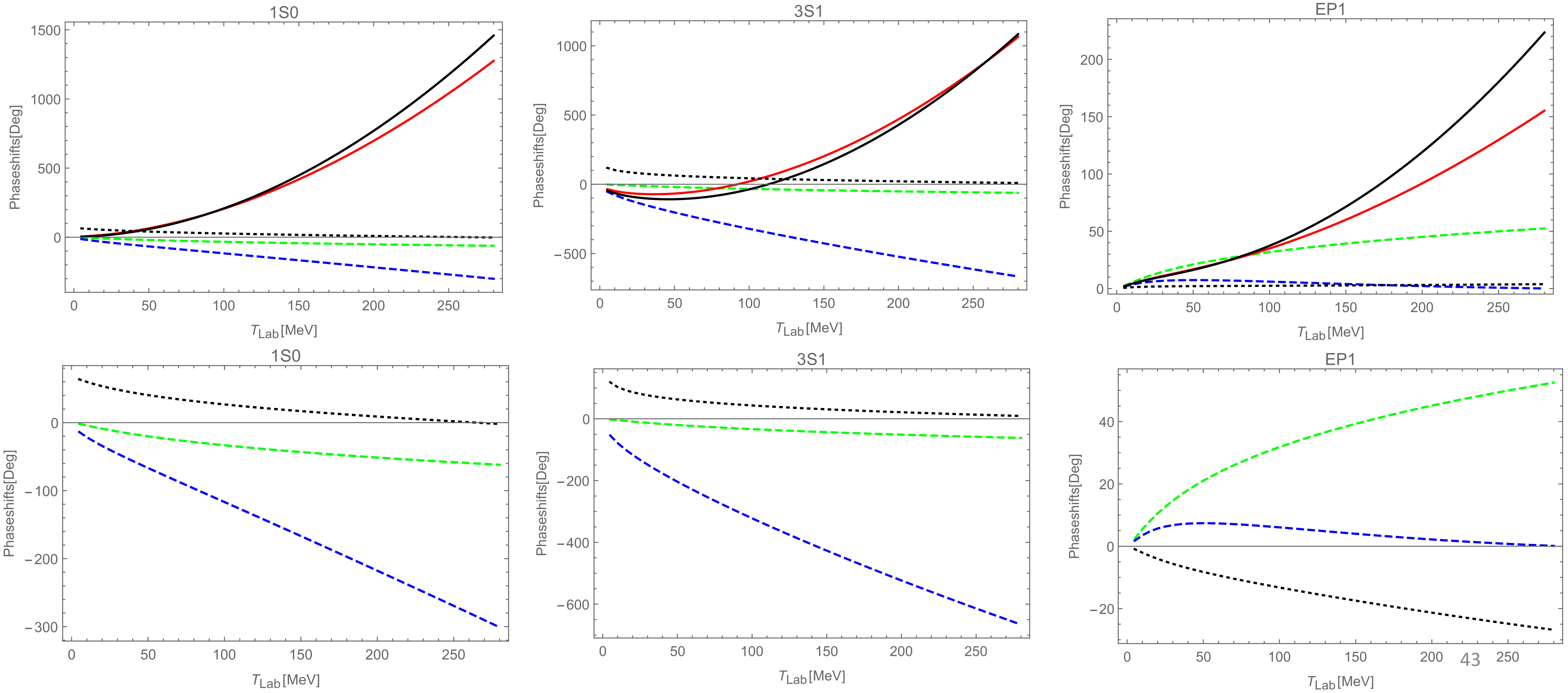
Numerical details

	g_A	f_π (GeV)	$c_1(\mathrm{GeV}^{-1})$	$c_2(\mathrm{GeV}^{-1})$	$c_3(\mathrm{GeV}^{-1})$	$c_4(\mathrm{GeV}^{-1})$
Covariant	1.29	0.0924	-1.39	4.01	-6.61	3.92
Non-Relativistic	1.29	0.0924	-0.9	~	-5.3	3.6

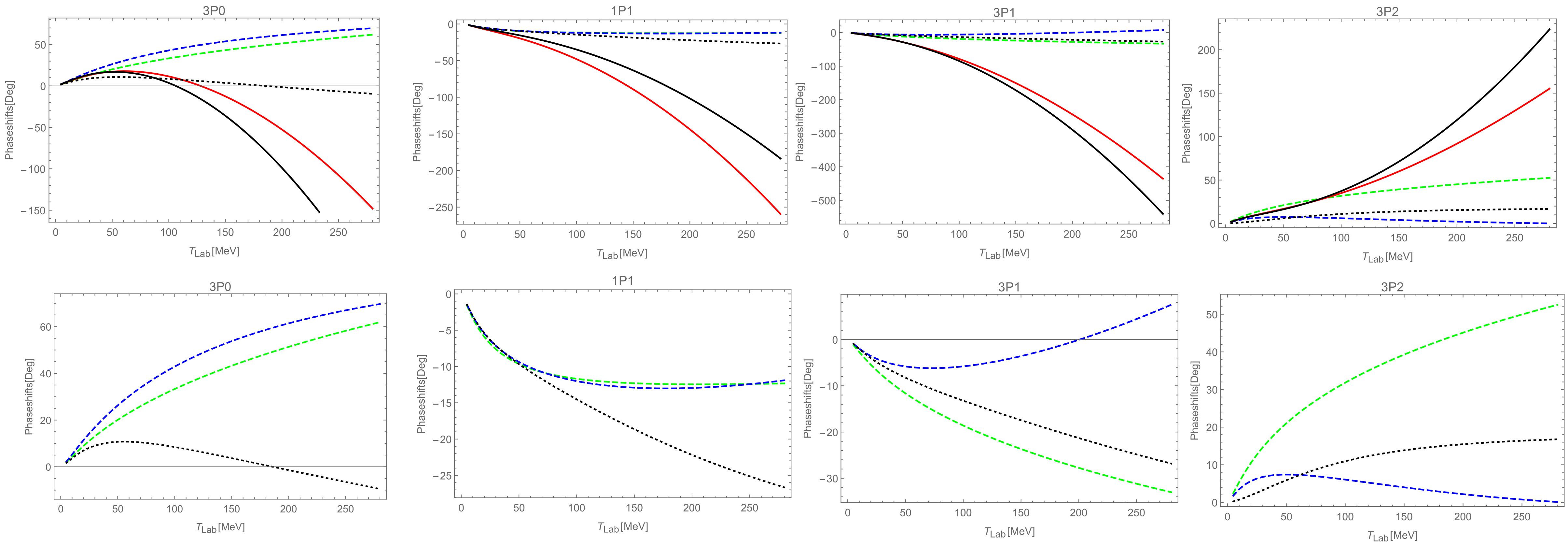
- Covariant LECs from Y. H. Chen, D. L. Yao, and H. Q. Zheng, PRD **87**, 054019 (2013).
- NR LECs from V. Bernard, N. Kaiser, and U. G. Meißner, NPA **615**, 483 (1997).

S wave TPE phase shifts (perturbative)

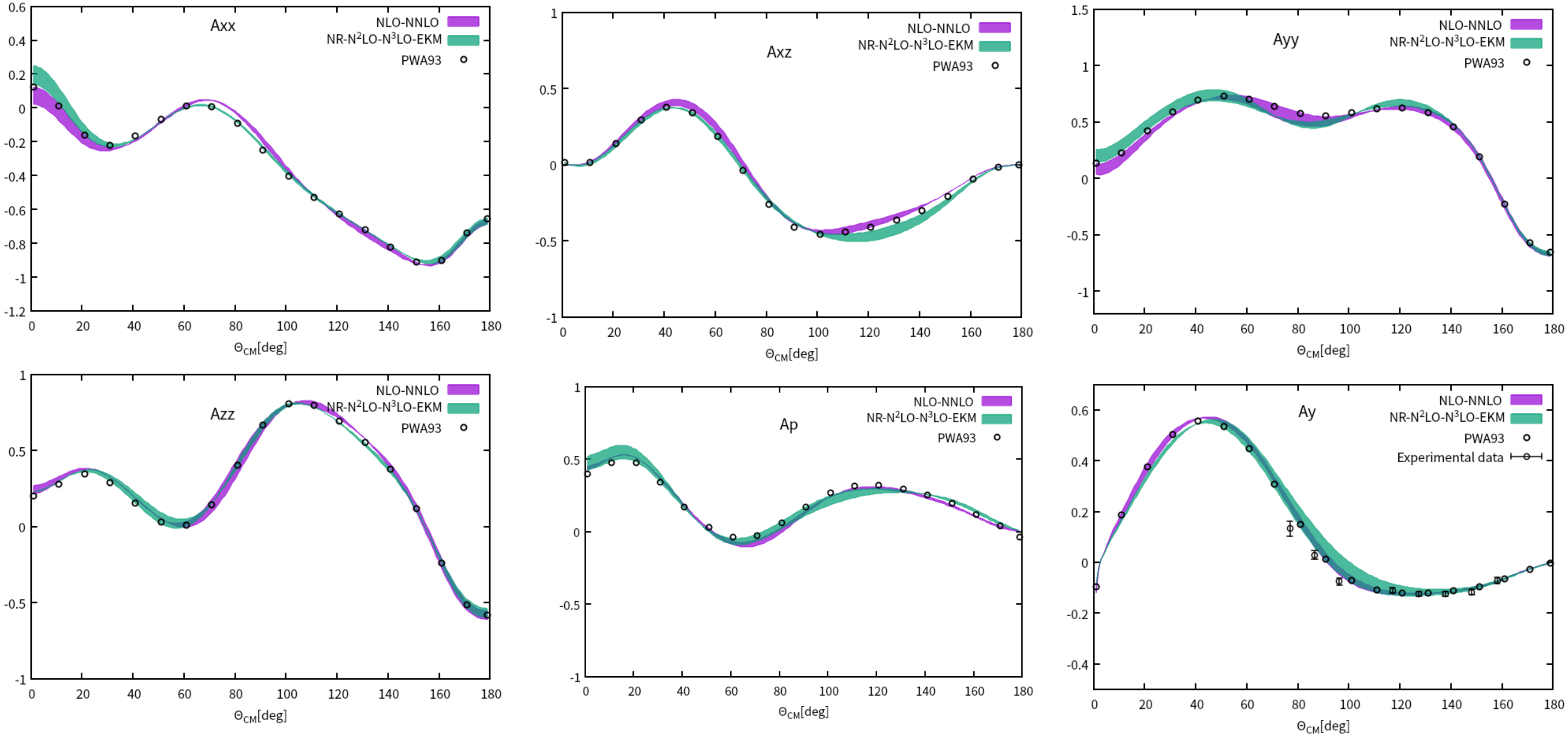




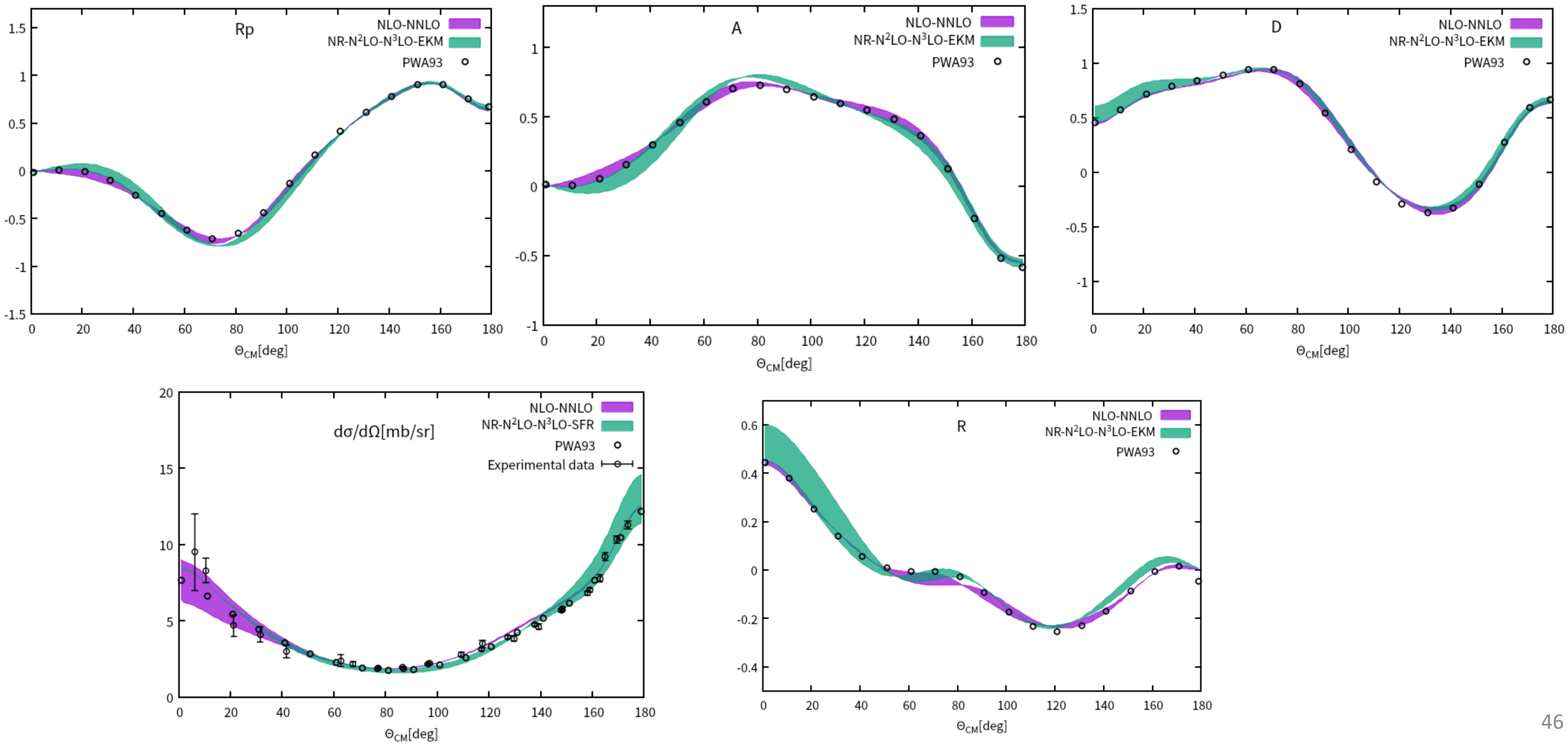
P wave TPE phase shifts



Observables



Observables



Cut off

	Regulator functions		Regulator exponent(s)	Chiral order/ cutoff range	$\pi N/$ 2π regularization	Fitting protocol
	Short (contact)	Long (pion exchanges)				
Local						
GT+ [22, 23]	$\alpha e^{-\tilde{r}^n}$	$1 - e^{-\tilde{r}^n}$	$n = 4$	Up to N^2LO $R_0 = 0.9 - 1.2$ fm	Fixed values from Ref. [26] SFR	Nijmegen PWA [27]
Semilocal						
EKM [9, 24]	$e^{-\tilde{p}^{n_1}} e^{-\tilde{p}'^{n_1}}$	$\left(1 - e^{-\tilde{r}^2}\right)^{n_2}$	$n_1 = 2$ $n_2 = 6$	Up to N^4LO $R_0 = 0.8 - 1.2$ fm $\Lambda \approx 493 - 329$ MeV	Fixed values [24] DR	Nijmegen PWA [27]
Nonlocal						
sim [25]	$e^{-\tilde{p}^{2n}} e^{-\tilde{p}'^{2n}}$	$e^{-\tilde{p}^{2n}} e^{-\tilde{p}'^{2n}}$	$n = 3$	Up to N^2LO $\Lambda = 450 - 600$ MeV	Fitting parameter in simultaneous fit SFR	Fits to NN, πN, and few-body systems $^{2,3}H$, ^3He
EMN [10]	$e^{-\tilde{p}^{2n_1}} e^{-\tilde{p}'^{2n_1}}$	$e^{-\tilde{p}^{2n_2}} e^{-\tilde{p}'^{2n_2}}$	$n_1 > \nu/2$ $n_2 = 2$ (4)	Up to N^4LO $\Lambda = 450 - 550$ MeV	Fixed values from Ref. [28] SFR	NN data from 1955-2016 [29]

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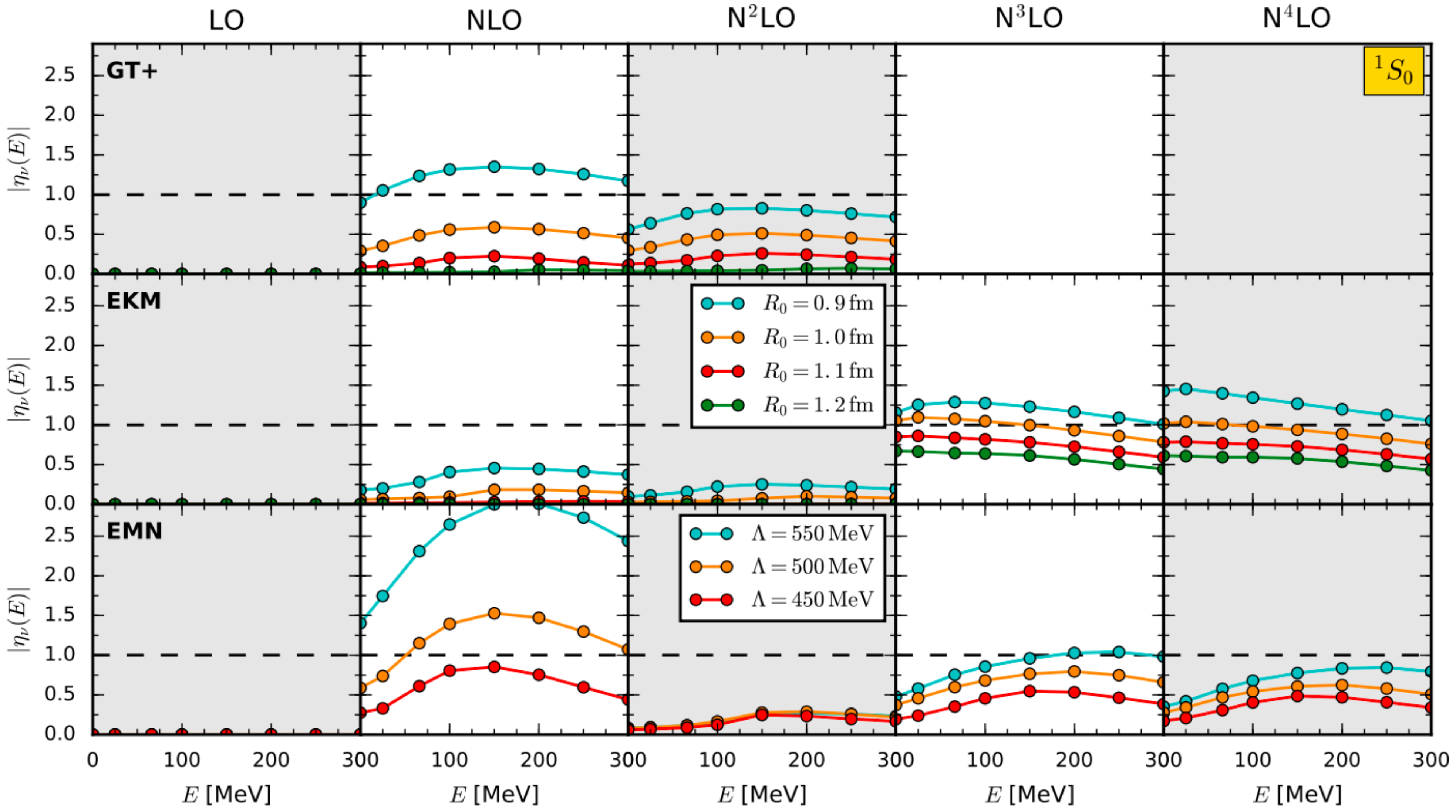
$$G_0(W)V\,|\Psi_\nu(W)\rangle = \eta_\nu(W)\,|\Psi_\nu(W)\rangle \;.$$

Weinberg eigenvalue



J. Hoppe et al., PRC 96 5, 054002 (2017)