



# 第一届“粤港澳”核物理论坛

## Ab initio no-core Gamow shell model calculations of Multineutron systems

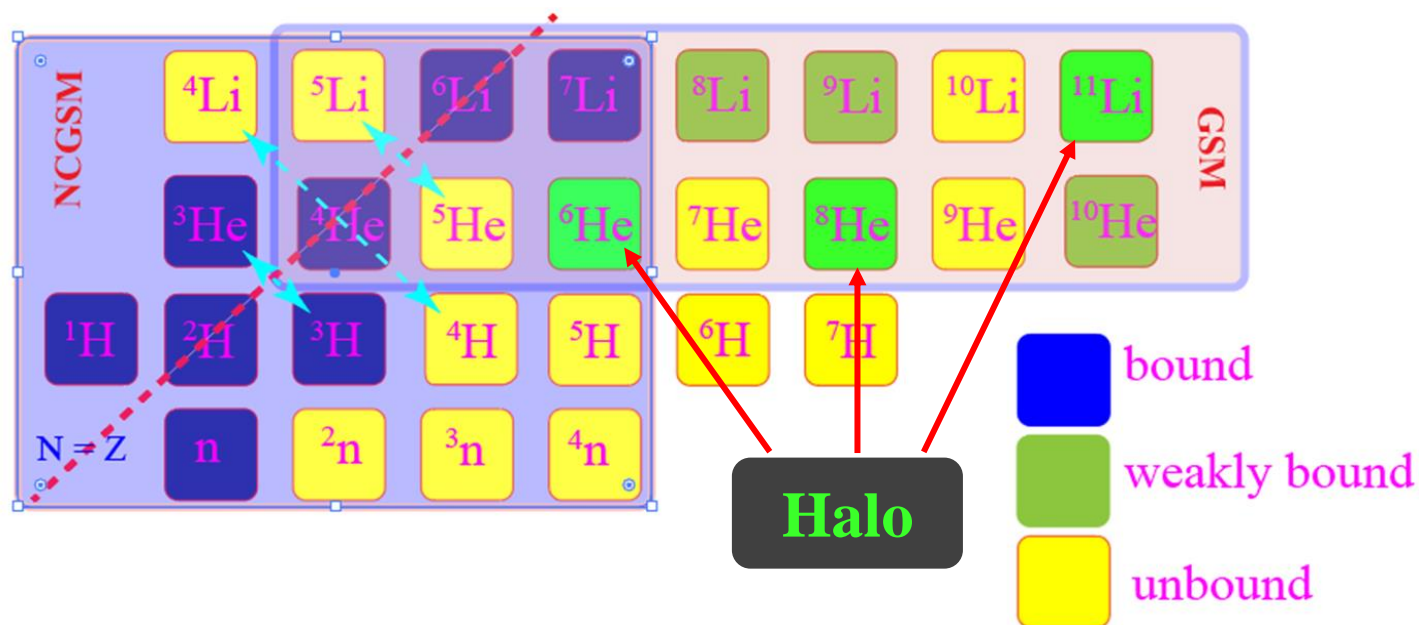
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Institute of Modern Physics, Chinese Academic of Science

In collaboration with

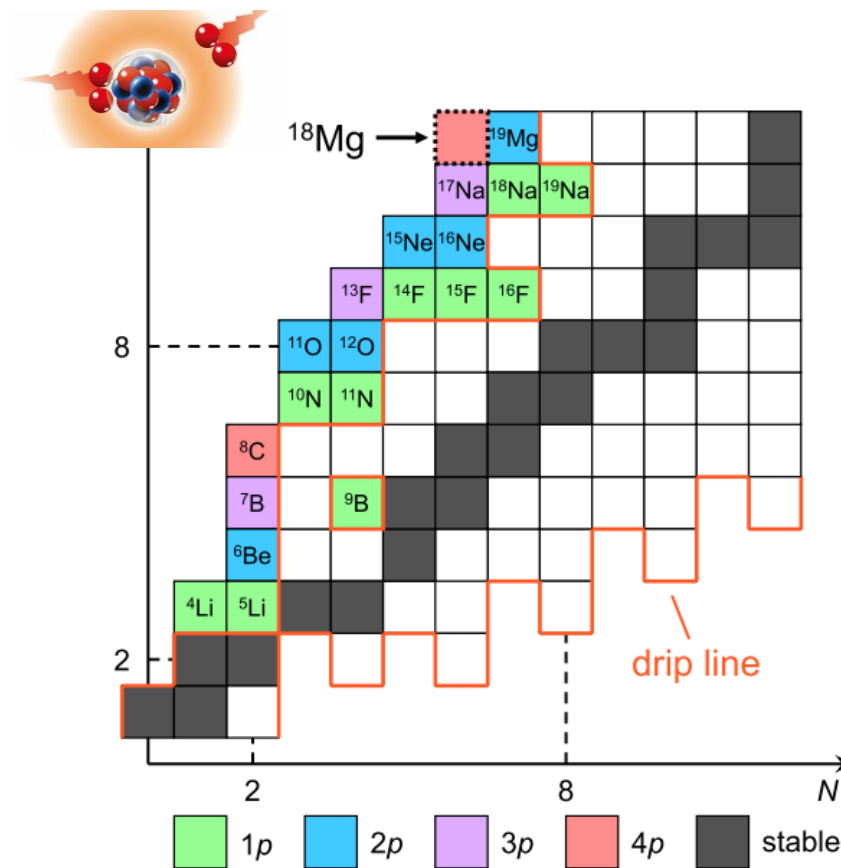
Furong Xu (许甫荣 教授), Nicolas Michel, Baishan Hu (胡柏山), and Wei Zuo (左维 研究员)

# Dripline nuclei



Exotic properties in dripline nuclei

- ✓ Halo structure  ${}^6\text{He}, {}^{11}\text{Li}, {}^{11}\text{Be}, {}^8\text{B}, {}^{17}\text{Ne}, {}^{29}\text{F}$
- ✓ Particle emission  $p, n, 2p, 2n, 4n, 2n+2n, \dots$
- ✓ Thomas Erhman-shift
- ✓ ...



*Y. Jin et al., Phys. Rev. Lett. 127, 262502 (2021).*

**Resonance and continuum couplings are important and need to be exactly treated!**

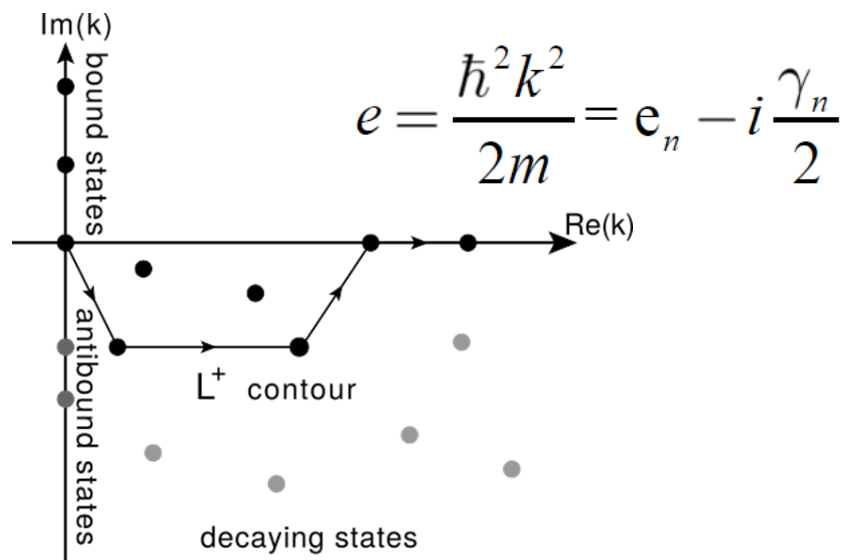
# Resonance state

A resonance state, which represents a decaying process, is time dependent.

Berggren complex- $k$  space transfers a time-dependent problem to a time-independent problem !

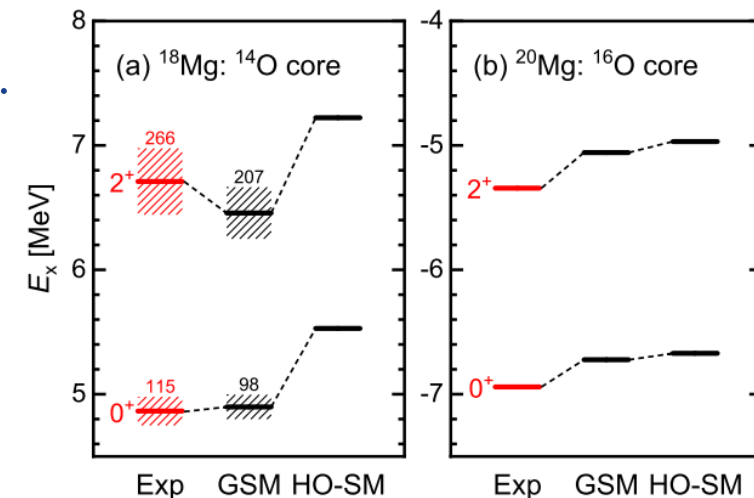
The radial wave function

$$\frac{d^2 u(k, r)}{dr^2} = \left( \frac{\ell(\ell+1)}{r^2} + \frac{2m}{\hbar^2} V(r) - k^2 \right) u(k, r)$$

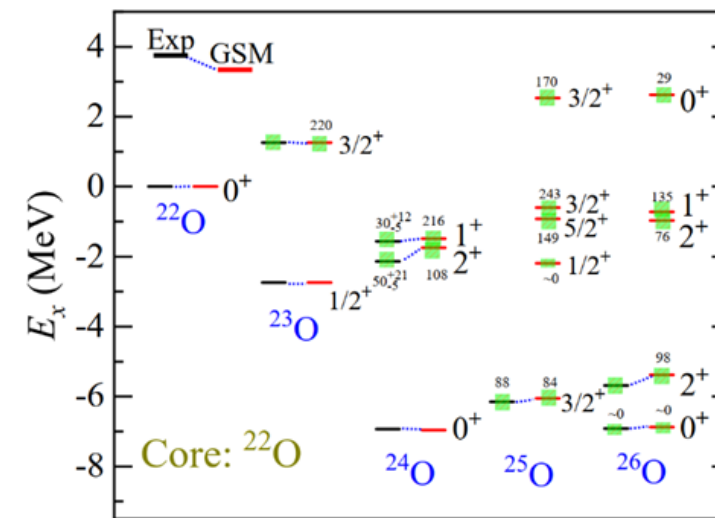


$$\sum_{n \in (b,d)} |u_n\rangle \langle u_n| + \int_{L^+} |u(k)\rangle \langle u(k)| dk = 1$$

T. Berggren, Nucl. Phys. A109 (1968) 265



Y. Jin et al., Phys. Rev. Lett. 127, 262502 (2021).



J. G. Li, et al. Phys. Rev. C 103, 034305 (2021)

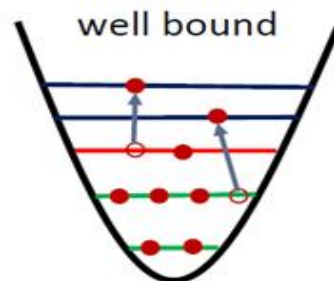
# Many-Body calculations in HO or Berggren basis

$$H = -\sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk}$$

NCSM/SM



Closed quantum system

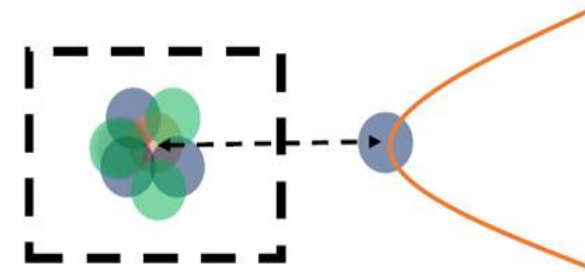


HO basis

*E. Caurier, et al. RMP 77,427 (2005)*  
*T. Otsuka, et al, RMP. 92,015002 (2020)*  
*B. R. Barrett, P. Navratil, and J. P. Vary PPNP 69,131(2013)*

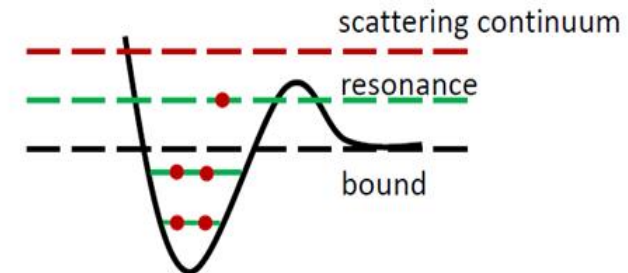
NCSM : also see Xingbo Zhao and Peng Ying's Presentations

NCGSM/GSM



Open quantum system

Including resonance and  
continuum coupling via using  
Berggren basis

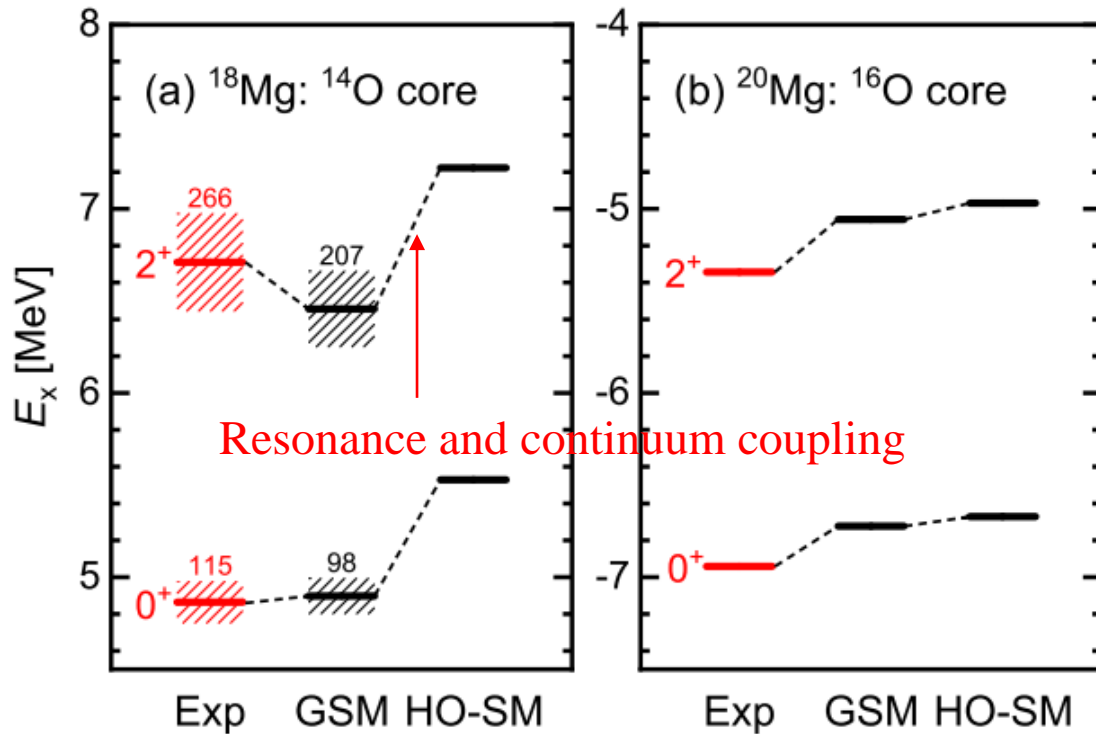


Berggren basis

*N. Michel, M. Płoszajczak, The Gamow Shell Model, Springer ;*  
*N. Michel, et al., JPG 36,013101 (2009)*  
*J. G. Li, et al., Physics 3, 977 (2021)*  
*J. G. Li, et al., Phys. Rev. C 100, 054313 (2019)*  
*J. G. Li, et al., Phys. Rev. C 104, 024319 (2021)*

# Resonance and continuum coupling in dripline nuclei :examples

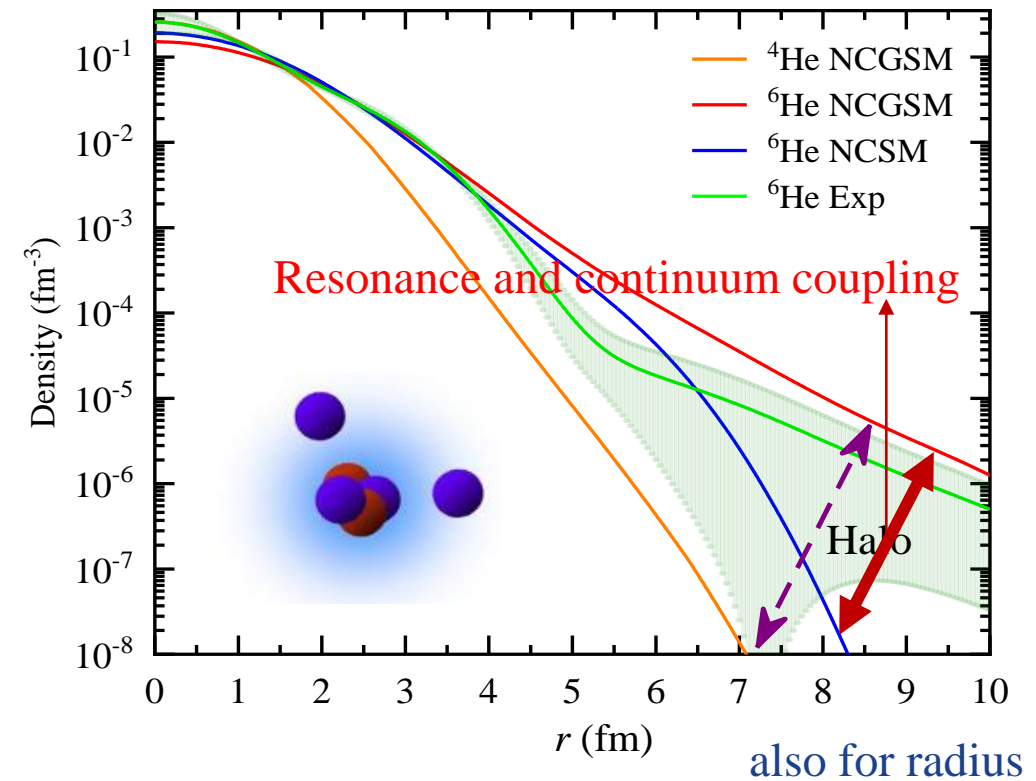
$^{18}\text{Mg}$  : 4p-decay to  $^{14}\text{O}$



Theoretical calculations are provided by **J. G. Li** and N. Michel

*Y. Jin et al., Phys. Rev. Lett. 127, 262502 (2021).*

$^6\text{He}$ : Two-neutron Halo



*J. G. Li, et al., in preparation*

# History of Multineutron system

- More than 50 years of multi-neutron searches, especially the tetraneutron.
  - See [arXiv:1608.00169](https://arxiv.org/abs/1608.00169)[nucl-th] (2016) Eur. Phys. J. A 57,105(2021),
- Earlier experiments gave negative results, no information about the multi-neutron systems.
- In 2002, Marqués *et al* reported the possible existence of a **bound** tetraneutron observed in a breakup reaction of the  $^{14}\text{Be} \rightarrow ^{10}\text{Be} + 4\text{n}$  channel. Marqués *et al.*, PRC 65, 044006 (2002)
- In 2016, Kisamori *et al* observed of few events of tetraneutron in the doubly charge-exchange reaction  $^4\text{He}(^8\text{He}, ^8\text{Be})$ . Interpreted the tetraneutron as a candidate **resonance** with  $E_r = 0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst}) \text{ MeV}$  and width  $\Gamma \leq 2.6 \text{ MeV}$ . Kisamori *et al.*, PRL 116, 044006 (2016)

reaction	initial state	final state	$\sigma$	results
$^4\text{He}(^8\text{He}, \alpha\alpha)^4\text{n}$ □ Shimoura, NP1512-SHARAQ10			nb	$N_{\text{evt}} \sim 10\text{s}$ $^4\text{n} : E, \Gamma$
$^8\text{He}(p, p\alpha)^4\text{n}$ □ Paschalis, NP1406-SAMURAI19			$\mu\text{b}$	$N_{\text{evt}} \sim 1000\text{s}$ $^4\text{n} : E, \Gamma$
$^8\text{He}(p, 2p)\{^3\text{H} + ^4\text{n}\}$ □ FMM/Yang, NP1512-SAMURAI34			mb	$N_{\text{evt}} \sim 10,000\text{s}$ $^4\text{n} \& ^3\text{H} : E, \Gamma, \Omega$

Nature 2022

Future tetraneutron experiments

# History of theoretical researches for the multi-neutron systems

Year	Author	Journal	Conclusion
2003	Bertulani et al	J. Phys. G 29, 2431	gave no bound $4n$ combined dineutron-dineutron molecule and a toy NN potential
2003	Steven C. Pieper	PRL. 90, 252501	employed the GFMC calculated the tetraneutron, showed the modern nuclear force can not tolerate a bound tetraneutron and suggested the a tetraneutron resonance near 2MeV
2005	Lazauskas and Carbonell	PRC 72, 034003	used Complex scaling based on Reid 93 NN potential: no low-lying $4n$ resonances : no low-lying tetraneutron resonance.
→ 2016 experiment			
2016	E.Hiyama <i>et al</i>	PRC <b>93</b> , 044004	employed Complex scaling using AV8'+(toy)NNN, low $4n$ resonance possible only by strongly strongly modify the nuclear force
2016	A.M.Shirokov <i>et al</i>	PRL <b>117</b> ,182502	performed the NCSM with JISP16 interaction conformed a resonant state in tetraneutron around 0.8 MeV, width 1.4 MeV
2017	S. Gandofi <i>et al</i>	PRL <b>118</b> , 232501(2017)	presented the QMC calculations of multi-neutron systems, suggested the trineutron and tetraneutron were both resonance

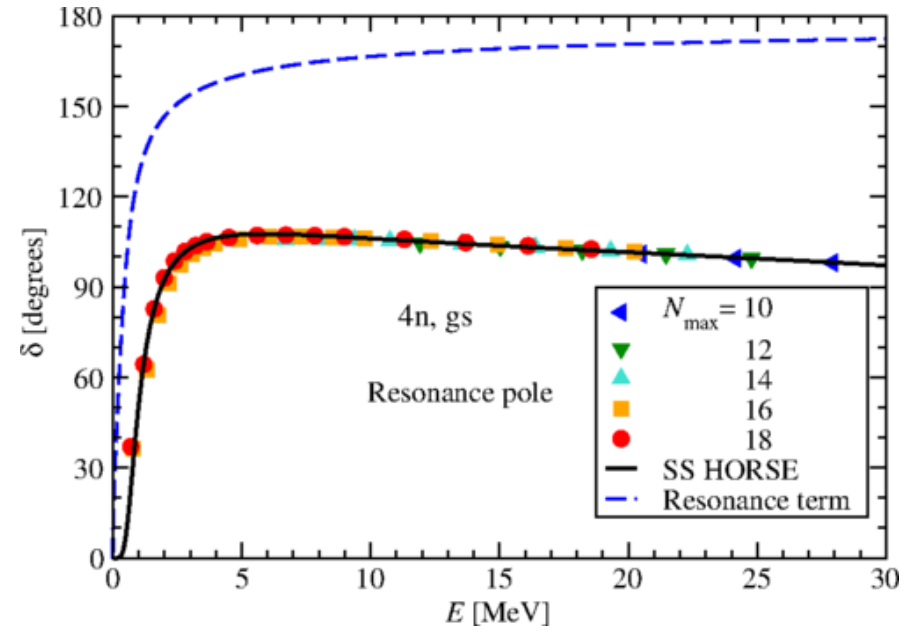
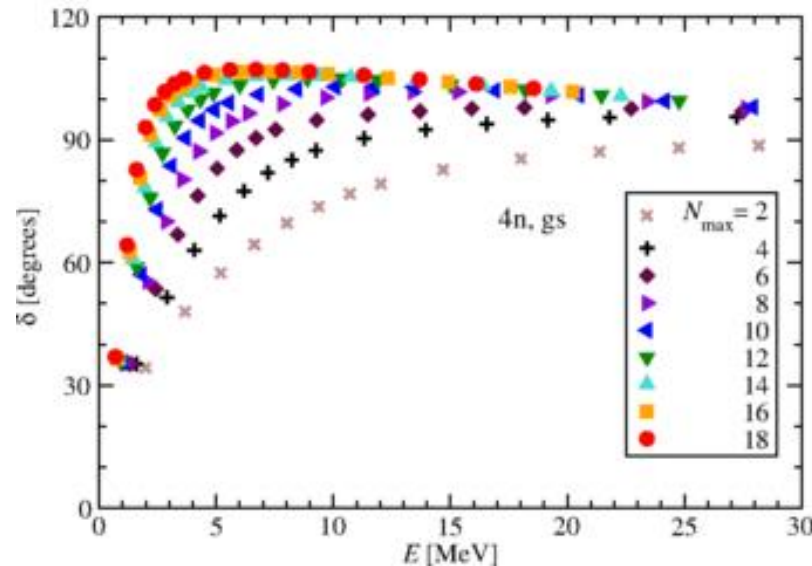
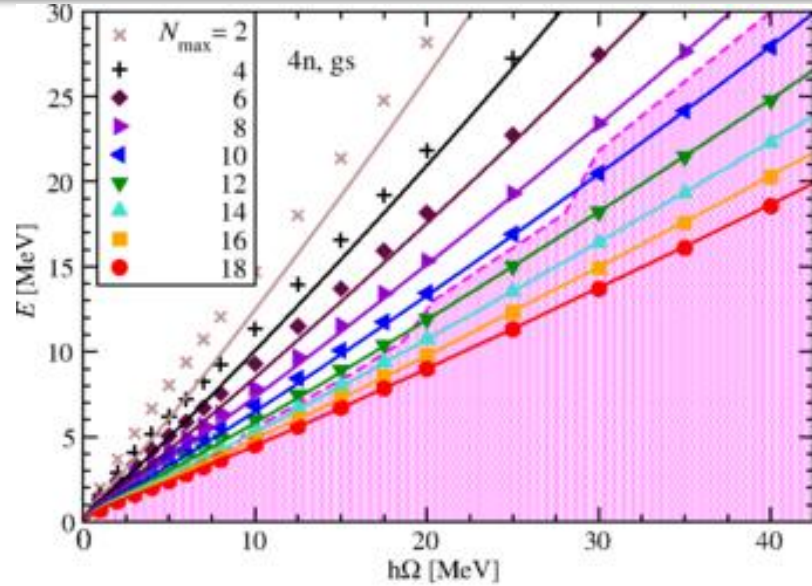


# History of theoretical researches for the multi-neutron systems

Year	Author	Journal	Conclusion
2017	K. Fosseze <i>et al.</i>	PRL <b>119</b> , 032501	performed NCGSM gave energy of tetraneutron may be compatible with experimental value, but the width must be too large
2018	A.Deltuva	PRC <b>97</b> , 034001 (2018), PLB <b>782</b> , 238 (2018)	employed Faddeev method gave the absence of an observable trineutron and tetraneutron resonance based on modern two-body force
2018	A.M.Shirokov <i>et al.</i>	AIP Conf. proc 020038	Performed NCSM for tetraneutron with different two-body force, similar results are obtained
2019	A.M.Shirokov <i>et al.</i>	Presentation in Nanjing@China 2019	updated their calculations and gave two resonance states in tetraneutron
2019	J. G. Li <i>et al.</i>	PRC 100 054313	Performed NCGSM for trineutron and tetraneutron, predicting that $E(^3n) = 1.29 \text{ MeV}$ $\Gamma(^3n) = 0.91 \text{ MeV}$ $E(^4n) = 2.64 \text{ MeV}$ $\Gamma(^4n) = 2.38 \text{ MeV}$
2020 2021	Michael D. Higgins <i>et al.</i>	PRL 125,052501 PRC 103 024004	Using adiabatic hyperspherical framework, Predicting that that no resonance and no bound state exists for the tetraneutron system



# NCSM for tetra-neutron



$E(\text{tetra-neutron}) = 0.8 \text{ MeV}$     $\Gamma(\text{tetra-neutron}) = 1.4 \text{ MeV}$

*A. M. Shirokov, et al, PRL 117,182502 (2016)*

2019' workshop, they predict two resonance states of tetra-neutron:

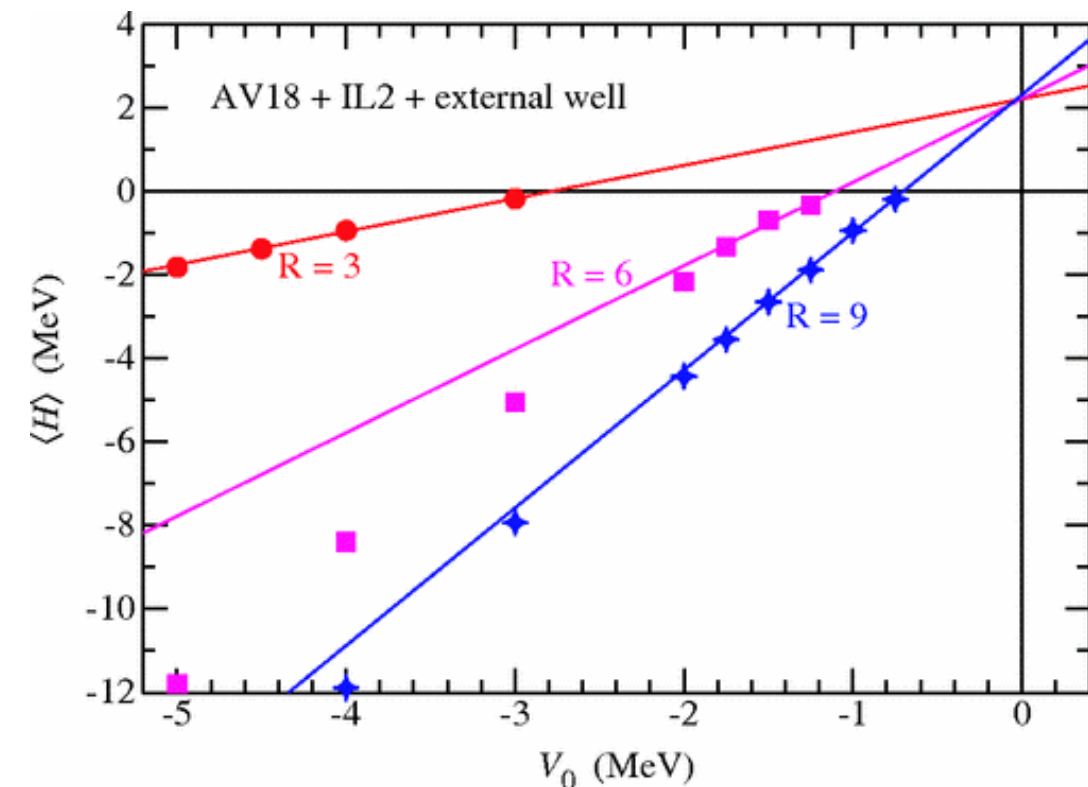
1<sup>st</sup>:  $E(4n) \sim 0.3 \text{ MeV}$     $\Gamma(4n) \sim 815 \text{ keV}$

2<sup>st</sup>:  $E(4n) \sim 0.8 \text{ MeV}$     $\Gamma(4n) \sim 1.3 \text{ MeV}$

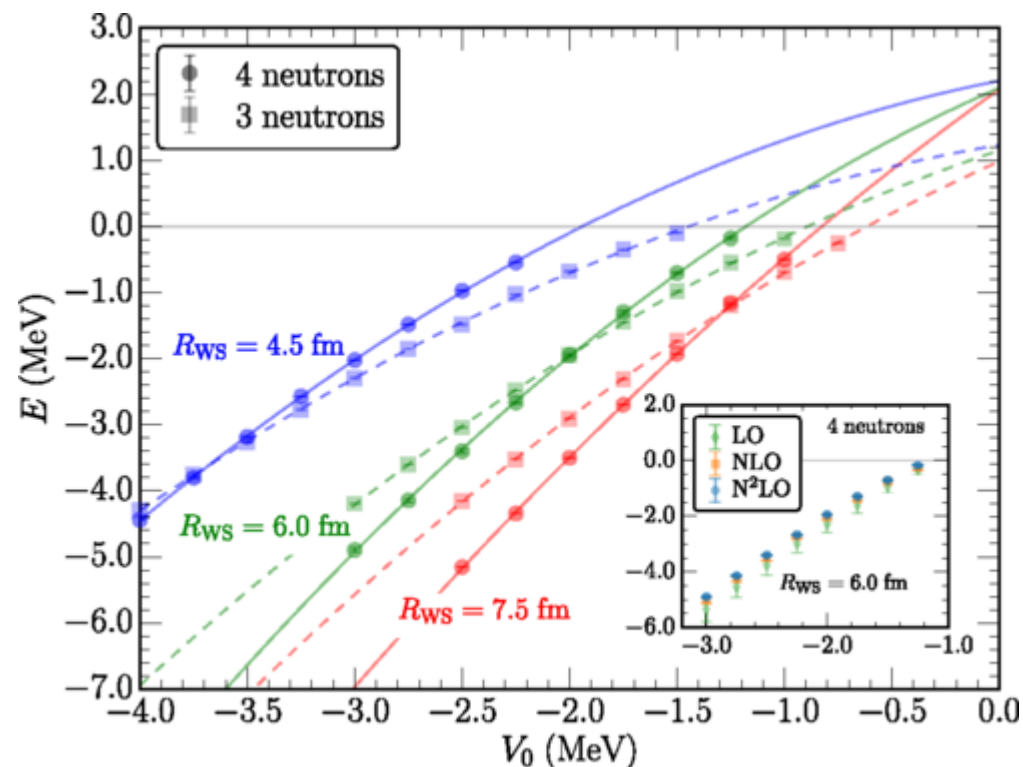
*A. M. Shirokov Presentation in Nanjing@China 2019*

# QMC calculations of multineutron systems

$$H = -\sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_i V_{ws}(r_i) + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$



Steven C. Pieper PRL 90, 252501(2002)

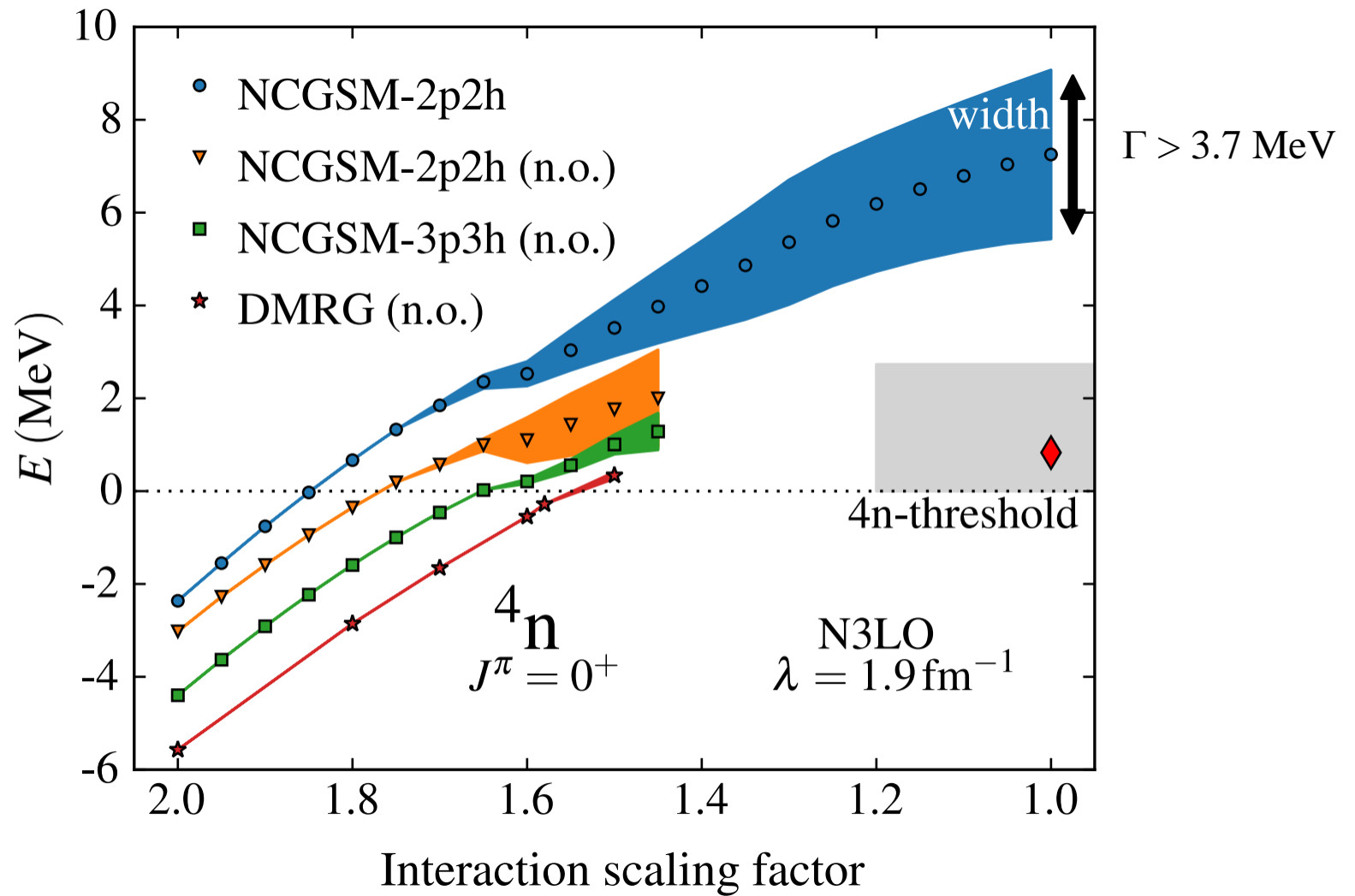


S. Gandolfi, et al., PRL 118, 232501(2017)

Only energy, no width!

# NCGSM calculations of multineutron systems

	$\lambda = 1.7 \text{ fm}^{-1}$	$\lambda = 1.9 \text{ fm}^{-1}$	$\lambda = 2.1 \text{ fm}^{-1}$
N3LO	7.27 (3.69)	7.28 (3.67)	7.28 (3.69)
N2LO <sub>opt*</sub>	7.32 (3.74)	7.33 (3.78)	7.34 (3.95)
N2LO <sub>sat</sub>	7.24 (3.48)	7.22 (3.58)	7.27 (3.55)
JISP16		7.00 (3.72)	



*K. Fosse, J. Rotureau, N. Michel, and M. Ploszajczak, PRL 119, 032501 (2017)*

# NCGSM for multi-neutron systems

Interaction : N3LO  $V_{\text{low-}k}$   $2.1\text{fm}^{-1}$

Model space :  $s_{1/2}, p_{3/2}$  in Berggren basis (45 points)

$p_{1/2}, d_{5/2,3/2}, f_{5/2,7/2}, g_{9/2}$  in HO basis. ( $N_{\text{max}} \leq 20$ )

Method : Davison + overlap method

N. Michel, W. Nazarewicz, M. Ploszajczak, and T. Vertse, JPG 36, 013101 (2009)

Multineutron systems

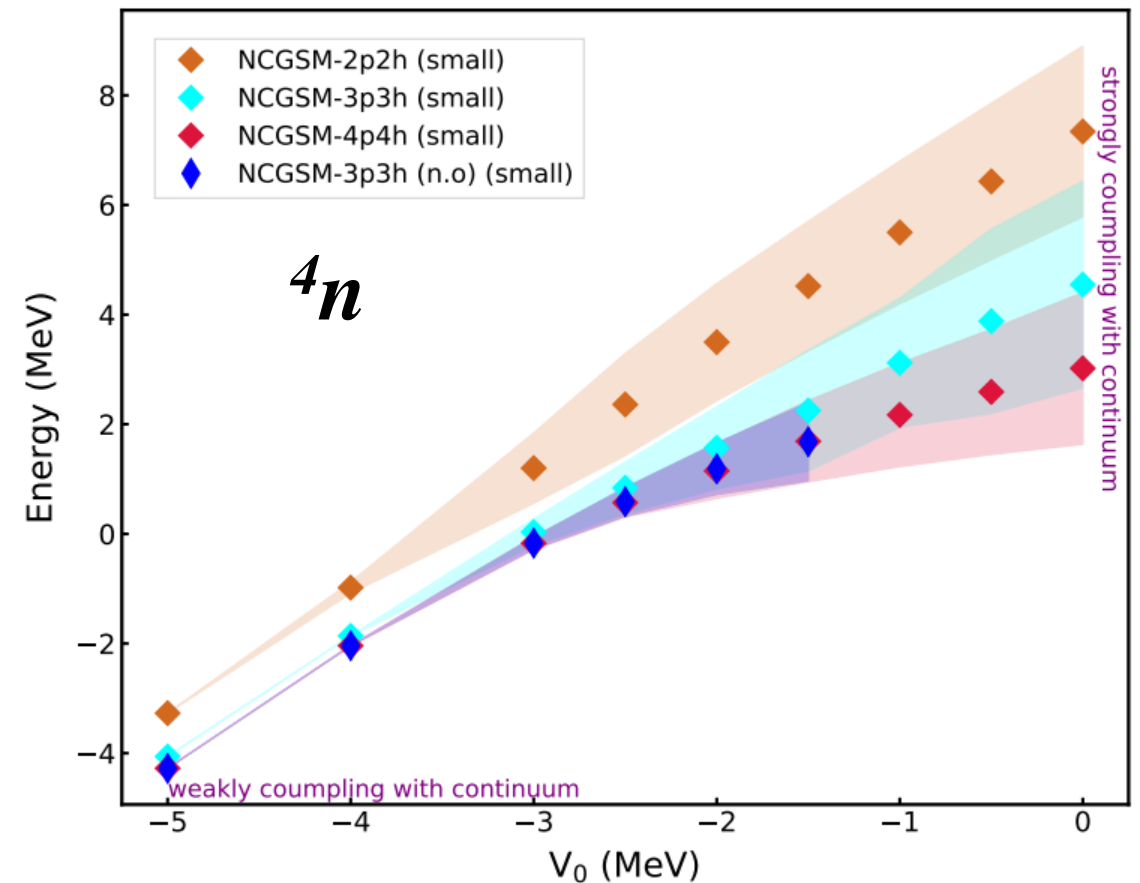
Unbound

Strongly coupling with continuum

$$H = \frac{1}{A} \sum_i^A \frac{(p_i - p_j)^2}{2m} + \sum_{i < j}^A V_{NN}^{i < j} + \sum_i^A V_{WS}$$

small model space :  $s_{1/2}, p_{3/2}$  in Berggren basis (45 points)

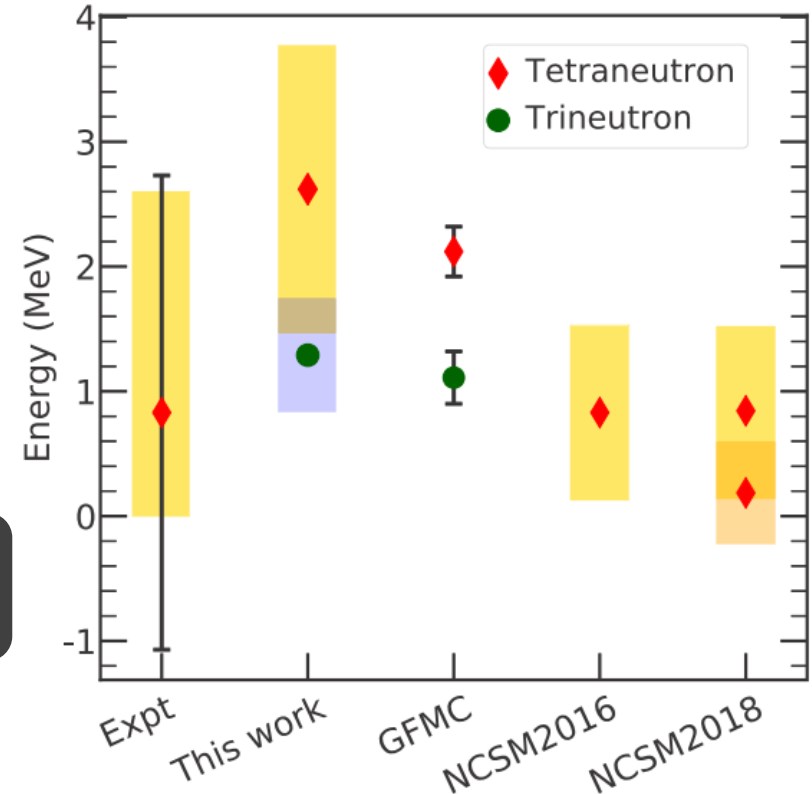
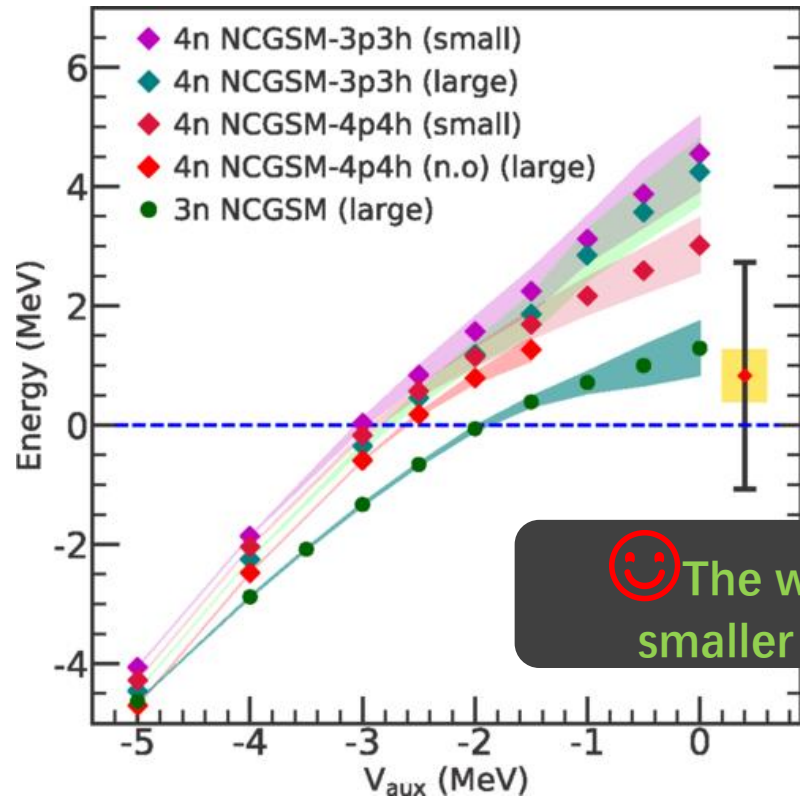
$p_{1/2}, d_{5/2,3/2}, f_{5/2,7/2}, g_{9/2}$  in HO basis. ( $N_{\text{max}} \leq 4$ )



# NCGSM for multi-neutron systems

$$E(^3\text{n}) = 1.29 \text{ MeV} \quad \Gamma(^3\text{n}) = 0.91 \text{ MeV}$$
$$E(^4\text{n}) = 2.64 \text{ MeV} \quad \Gamma(^4\text{n}) = 2.38 \text{ MeV}$$

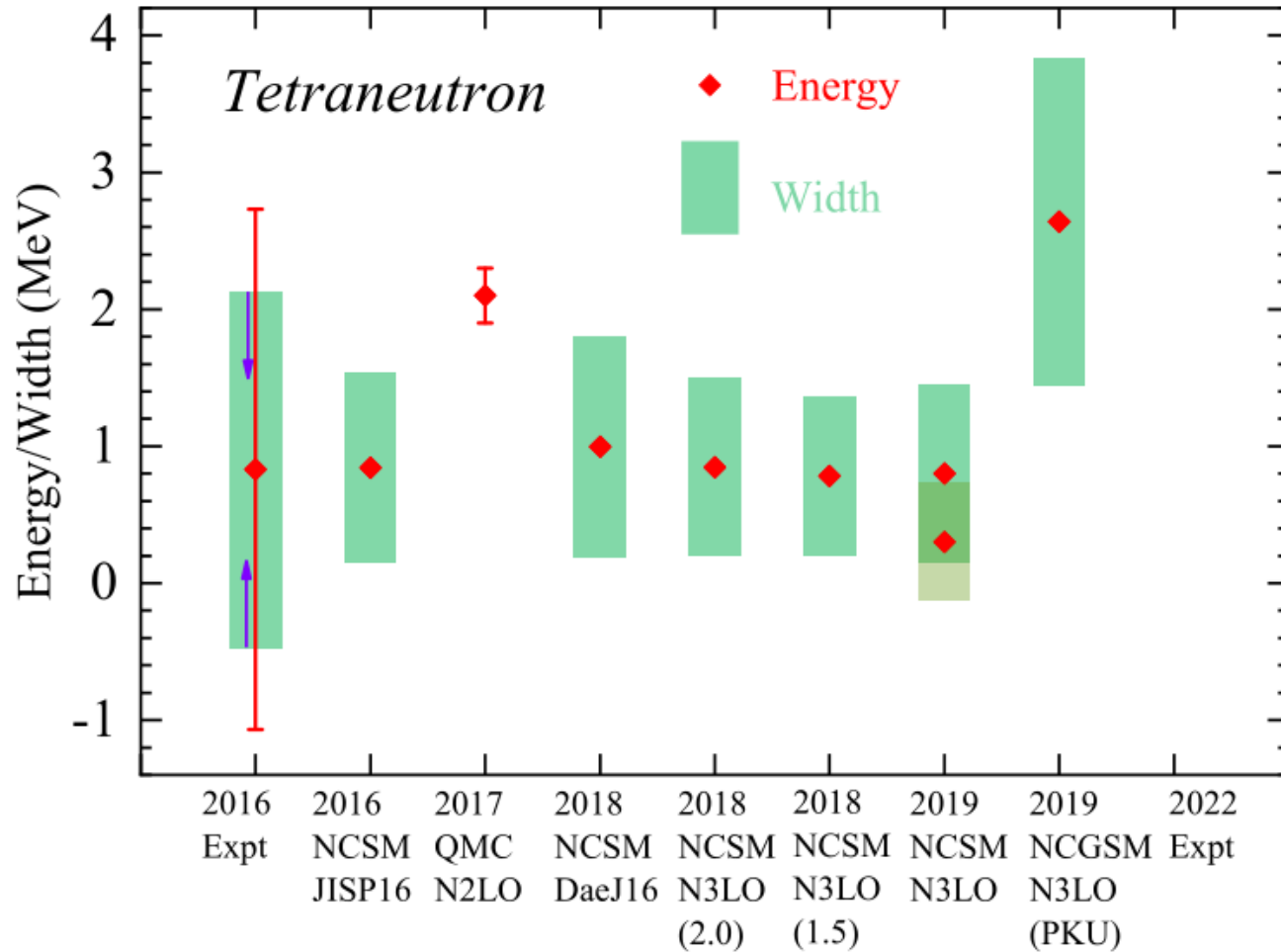
The energy of trineutron is lower than tetraneutron. QMC extrapolations agree with our calculations



☺ Trineutron Would be More Possible to be Observed than Tetraneutron

*J. G. Li, N. Michel, B. S. Hu, W. Zuo, and F. R. Xu\*, Phys. Rev. C 100, 054313 (2019)*

# The History of Tetraneutron



- ✓ 2016 Expt : *Kisamori et al., PRL 116, 044006 (2016)*
- ✓ 2016 NCSM: *A. M. Shirokov, et al, PRL 117, 182502 (2016)*
- ✓ 2017 QMC : *S. Gandolfi, et al., PRL 118, 232501(2017)*
- ✓ 2018 NCSM : *A. M. Shirokov, et al AIP Conf. proc 020038 (2018)*
- ✓ 2019 NCSM : *A. M. Shirokov Presentation in Nanjing@China 2019*
- ✓ **2019 NCGSM** : *J. G. Li, N. Michel, B. S. Hu, W. Zuo, and F. R. Xu\*, Phys. Rev. C 100, 054313 (2019)*



# Observation of a correlated free four-neutron system

## Article

## Observation of a correlated free four-neutron system

<https://doi.org/10.1038/s41586-022-04827-6>

Received: 4 August 2021

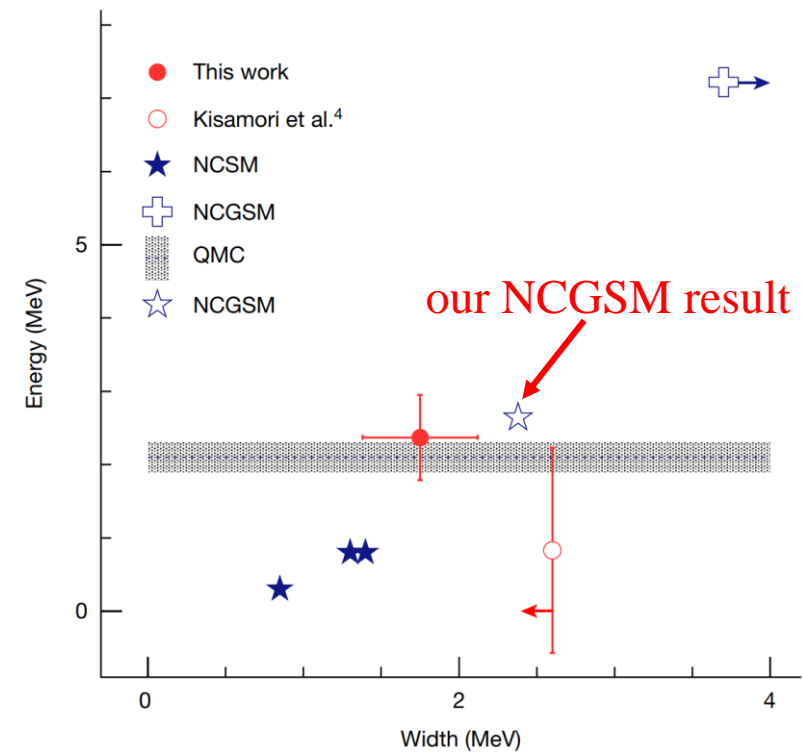
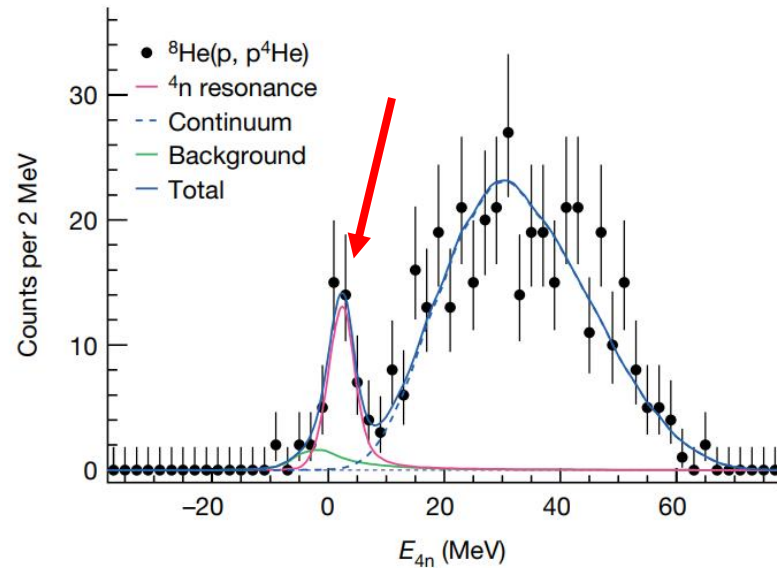
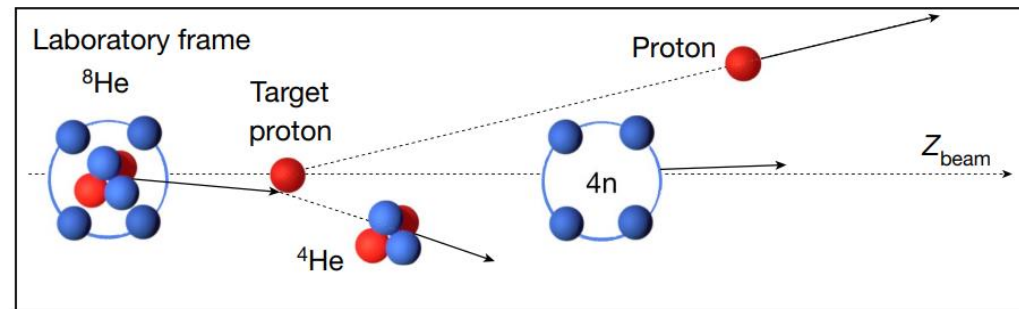
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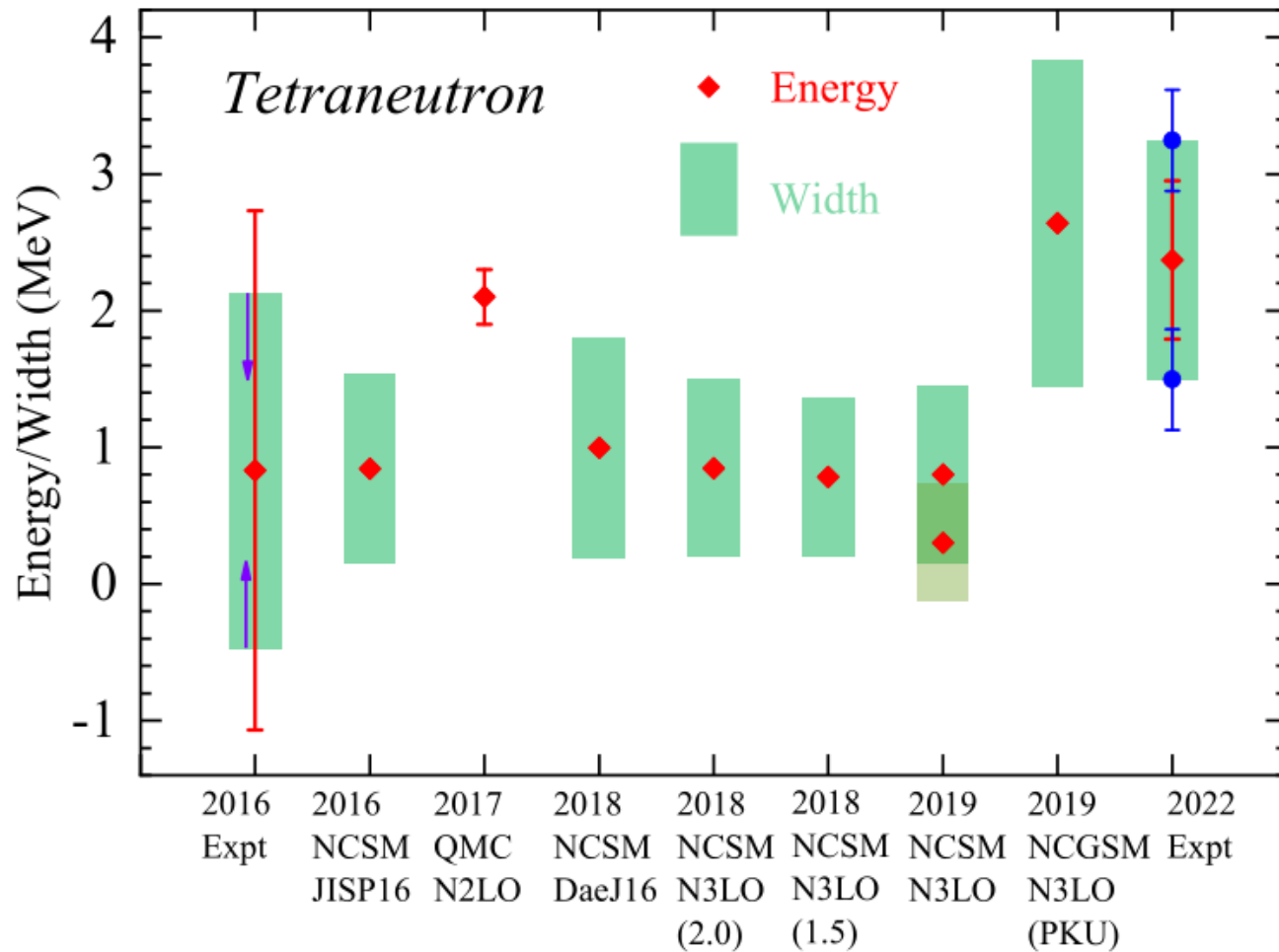
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M. Duer<sup>1,2,3</sup>, T. Aumann<sup>1,2,3</sup>, R. Gernhäuser<sup>4</sup>, V. Panin<sup>2,5</sup>, S. Paschalis<sup>1,6</sup>, D. M. Rossi<sup>1</sup>, N. L. Achouri<sup>7</sup>, D. Ahn<sup>5,16</sup>, H. Baba<sup>5</sup>, C. A. Bertulani<sup>8</sup>, M. Böhmer<sup>4</sup>, K. Boretzky<sup>2</sup>, C. Caesar<sup>1,2,5</sup>, N. Chiga<sup>5</sup>, A. Corsi<sup>9</sup>, D. Cortina-Gil<sup>10</sup>, C. A. Douma<sup>11</sup>, F. Dufter<sup>4</sup>, Z. Elekes<sup>12</sup>, J. Feng<sup>13</sup>, B. Fernández-Domínguez<sup>10</sup>, U. Forsberg<sup>6</sup>, N. Fukuda<sup>5</sup>, I. Gasparic<sup>1,5,14</sup>, Z. Ge<sup>5</sup>, J. M. Gheller<sup>9</sup>, J. Gibelin<sup>7</sup>, A. Gillibert<sup>9</sup>, K. I. Hahn<sup>15,16</sup>, Z. Halász<sup>12</sup>, M. N. Harakeh<sup>11</sup>, A. Hirayama<sup>17</sup>, M. Holl<sup>1</sup>, N. Inabe<sup>5</sup>, T. Isobe<sup>5</sup>, J. Kahlbow<sup>1</sup>, N. Kalantar-Nayestanaki<sup>11</sup>, D. Kim<sup>16</sup>, S. Kim<sup>1,16</sup>, T. Kobayashi<sup>18</sup>, Y. Kondo<sup>17</sup>, D. Körper<sup>2</sup>, P. Koseoglou<sup>1</sup>, Y. Kubota<sup>5</sup>, I. Kuti<sup>12</sup>, P. J. Li<sup>19</sup>, C. Lehr<sup>1</sup>, S. Lindberg<sup>20</sup>, Y. Liu<sup>13</sup>, F. M. Marqués<sup>7</sup>, S. Masuoka<sup>21</sup>, M. Matsumoto<sup>17</sup>, J. Mayer<sup>22</sup>, K. Miki<sup>1,16</sup>, B. Monteagudo<sup>7</sup>, T. Nakamura<sup>17</sup>, T. Nilsson<sup>20</sup>, A. Obertelli<sup>19</sup>, N. A. Orr<sup>7</sup>, H. Otsu<sup>5</sup>, S. Y. Park<sup>15,16</sup>, M. Parlog<sup>7</sup>, P. M. Potlog<sup>23</sup>, S. Reichert<sup>4</sup>, A. Revel<sup>7,8,24</sup>, A. T. Saito<sup>17</sup>, M. Sasano<sup>5</sup>, H. Scheit<sup>1</sup>, F. Schindler<sup>1</sup>, S. Shimoura<sup>21</sup>, H. Simon<sup>2</sup>, L. Stuhl<sup>16,21</sup>, H. Suzuki<sup>5</sup>, D. Symochko<sup>1</sup>, H. Takeda<sup>5</sup>, J. Tanaka<sup>1,5</sup>, Y. Togano<sup>17</sup>, T. Tomai<sup>17</sup>, H. T. Törnqvist<sup>12</sup>, J. Tscheuschner<sup>1</sup>, T. Uesaka<sup>5</sup>, V. Wagner<sup>1</sup>, H. Yamada<sup>17</sup>, B. Yang<sup>15</sup>, L. Yang<sup>21</sup>, Z. H. Yang<sup>5</sup>, M. Yasuda<sup>17</sup>, K. Yoneda<sup>5</sup>, L. Zanetti<sup>1</sup>, J. Zenihiro<sup>5,25</sup> & M. V. Zhukov<sup>20</sup>



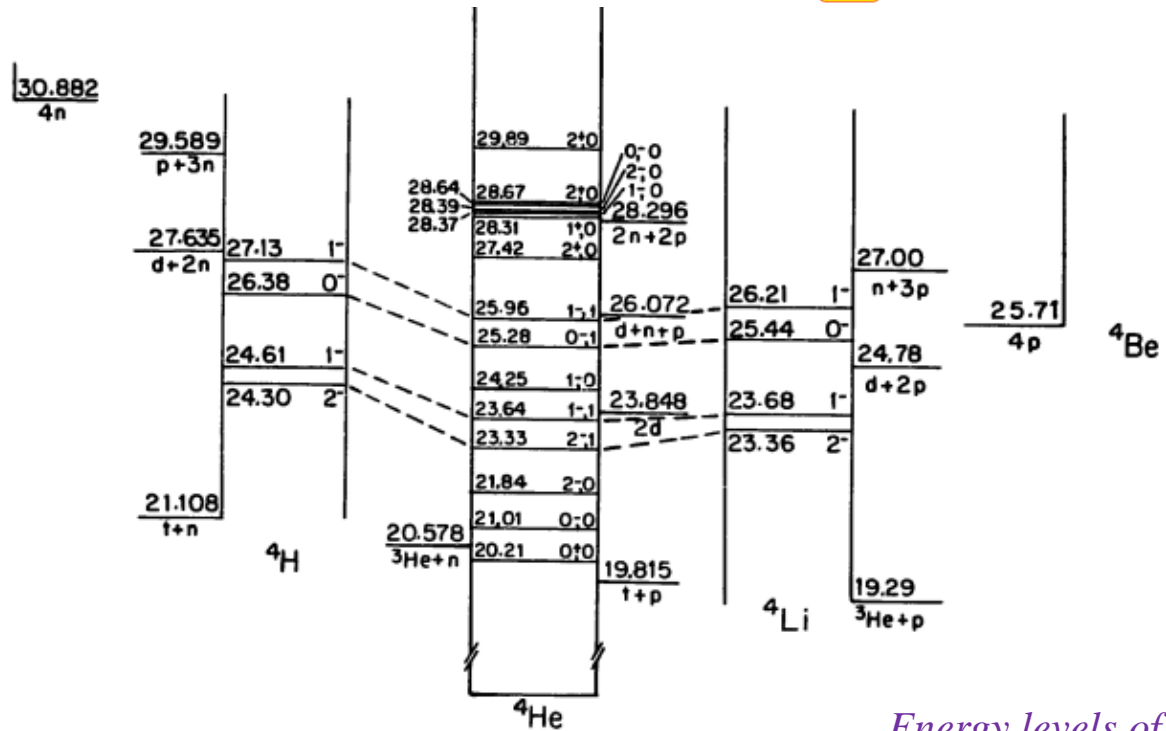
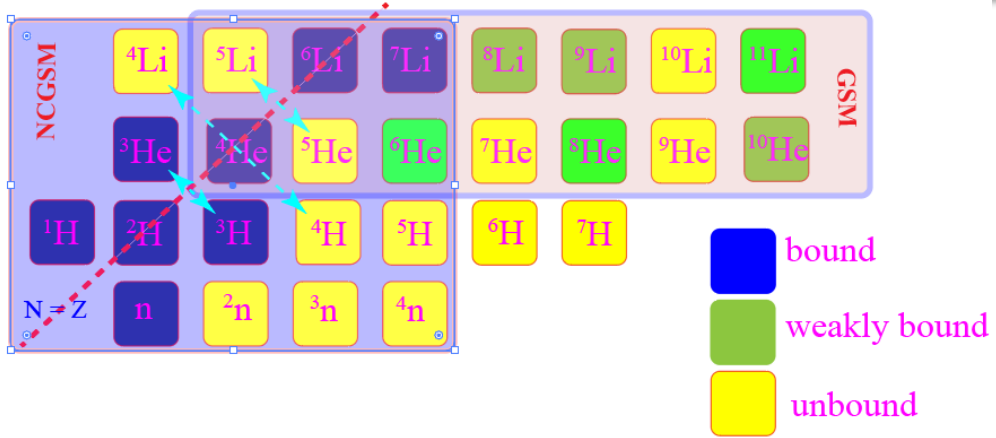


# The History of Tetraneutron



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- ✓ 2016 NCSM: *A. M. Shirokov, et al, PRL 117, 182502 (2016)*
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- ✓ 2019 NCGSM : *J. G. Li, N. Michel, B. S. Hu, W. Zuo, and F. R. Xu\*, Phys. Rev. C 100, 054313 (2019)*
- ✓ 2022 Expt : *M. Duer et al. Nature 606, pages678–682 (2022)*

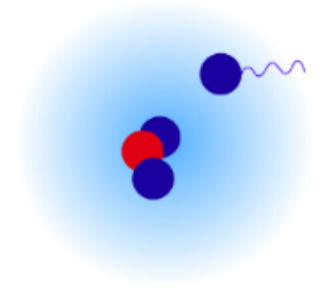
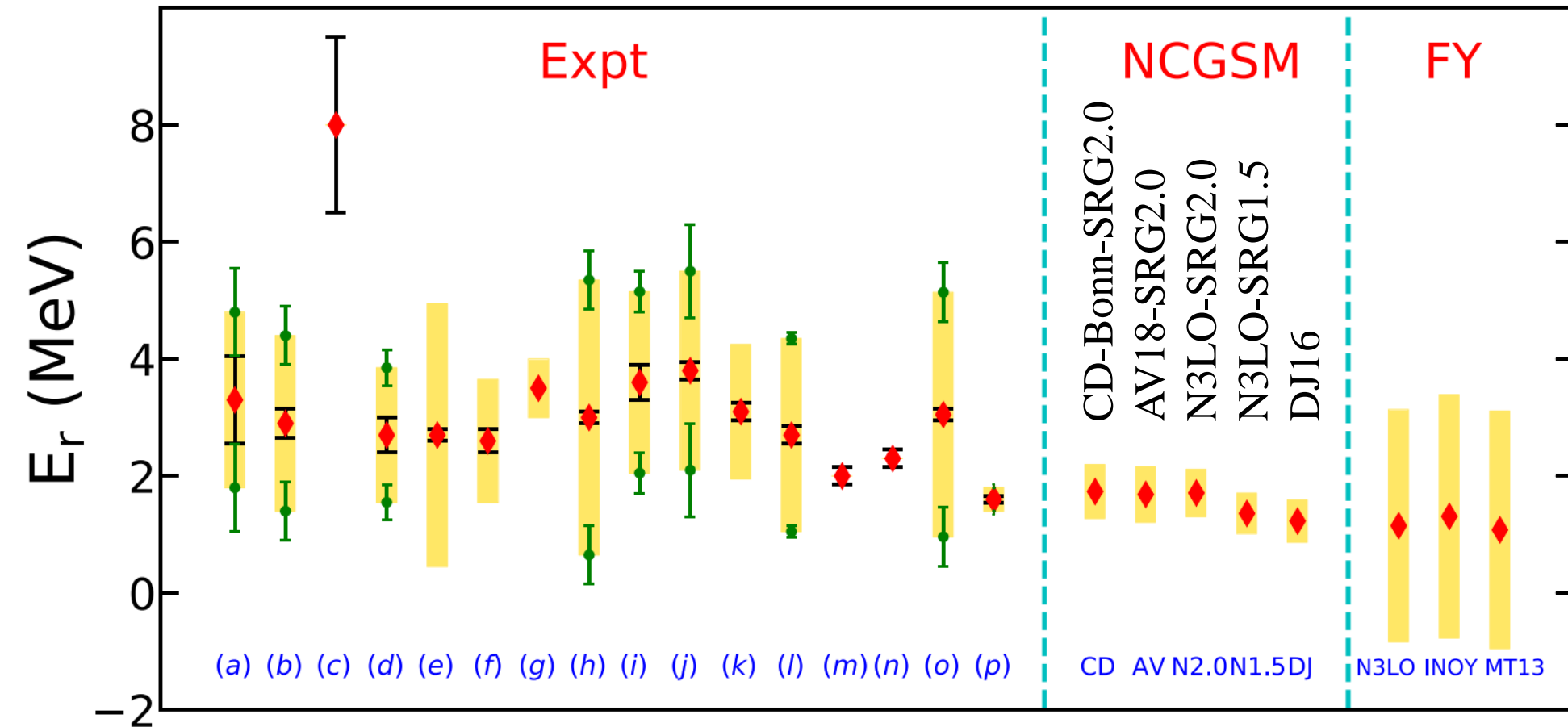
# $A = 4$ ( ${}^4\text{H}$ , ${}^4\text{He}$ and ${}^4\text{Li}$ ) nuclear systems



Energy levels of light nuclei  $A = 4$  D.R. Tilley et al. Nucl. Phys. A541 (1992) 1-104

- ✓  ${}^4\text{Li}$ ,  ${}^4\text{He}$  and  ${}^4\text{H}$ , all the states, except the ground state of  ${}^4\text{He}$ , are unbound bearing broad widths.
- ✓ The experimental energies and widths of the ground state of  ${}^4\text{H}$  and  ${}^4\text{Li}$  are either not very conclusive or even contraction.
- ✓ The  $T = 1$  isospin multiplet states: the ground states of  ${}^4\text{Li}$  and  ${}^4\text{H}$ , and the excited state of  ${}^4\text{He}$  → Isospin symmetry breaking

# NCGSM calculation of ${}^4\text{H}$

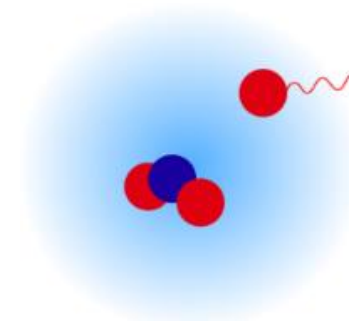
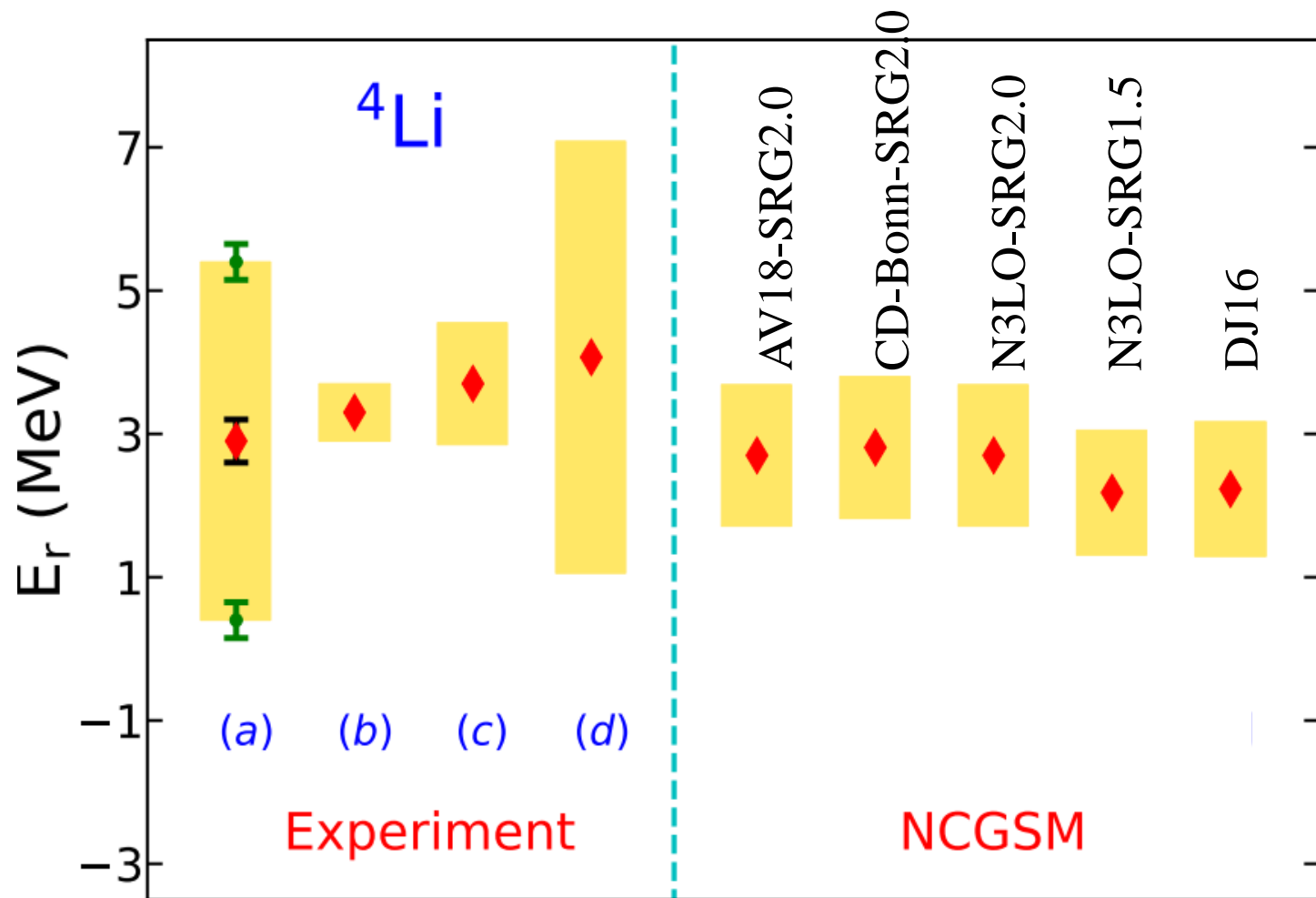


No coulomb force.

$E_r \sim 1.23\text{-}1.74$  MeV  
 $\Gamma \sim 725\text{-}951$  keV

*J.G. Li, N. Michel, W. Zuo and F.R. Xu\*, Phys. Rev. C 104, 024319 (2021)*

# *Ab initio* NCGSM calculation of ${}^4\text{Li}$



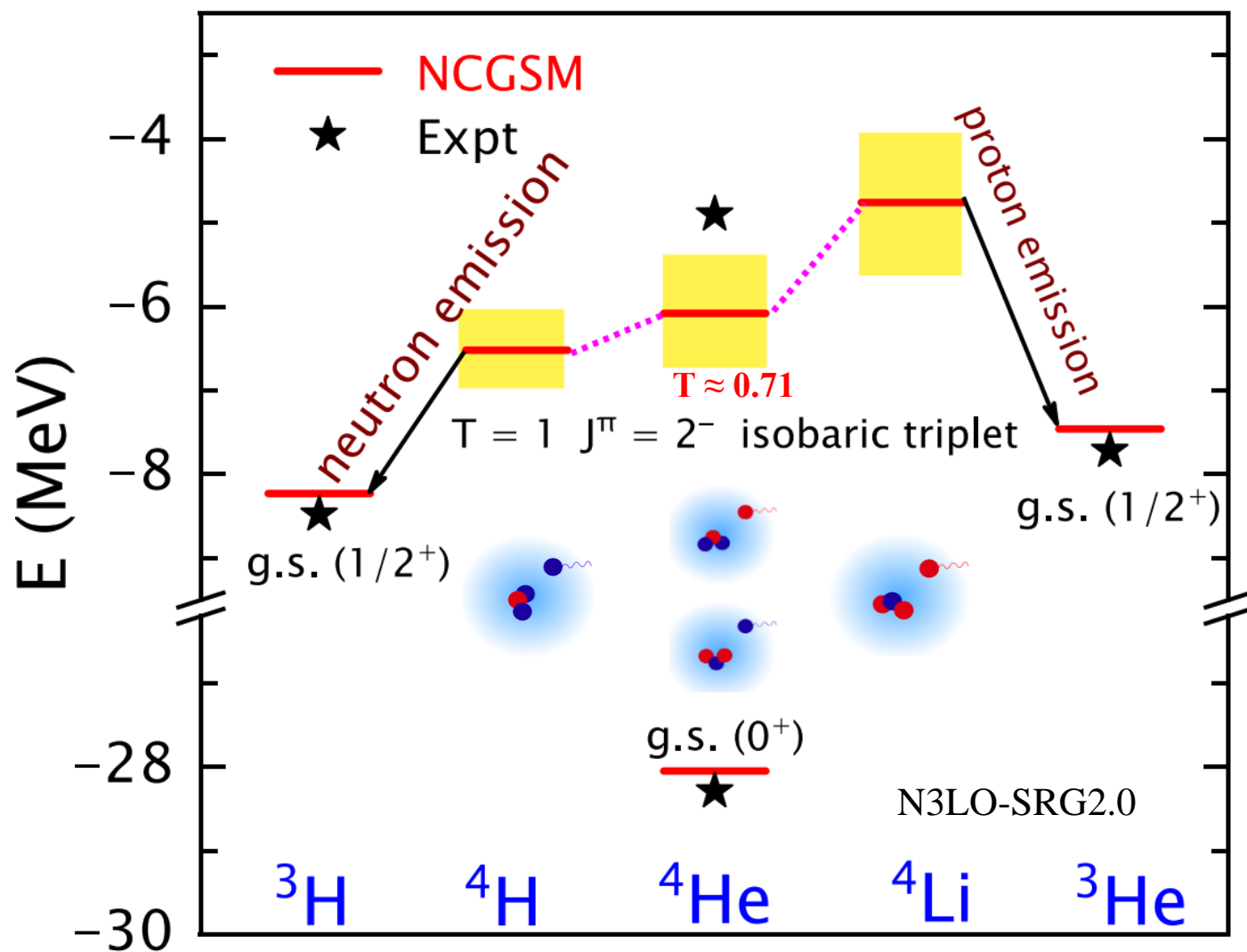
Coulomb force (long-range) need to be well treated.

$E_r \sim 2.23\text{-}2.70$  MeV

$\Gamma \sim 1881\text{-}1972$  keV

*J.G. Li, N. Michel, W. Zuo and F.R. Xu\*, Phys. Rev. C 104, 024319 (2021)*

# $A = 4$ ( ${}^4\text{H}$ , ${}^4\text{He}$ and ${}^4\text{Li}$ ) $T = 1$ state



- ✓ Comparing the calculated spectra of the  $A = 4$  systems, the ground states  ${}^4\text{Li}$  ( $T=1$ ),  ${}^4\text{H}$  ( $T=1$ ) ( $2^-$  and  $1^-$ ) with the ground state of  ${}^4\text{He}$  ( $T=0$ ) and  $2^-$  ( $T=1$ ) excited states.
- ✓ The isospin breaking is mainly caused by the Coulomb force.
- ✓ The energies and widths are both different due to the isospin breaking.
- ✓ The splitting of the  $T=2$  isobaric triplet states.

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# Summary

1. The Energy and Width of the Multi-neutron systems (Trineutron and Tetraneutron) are calculated with *ab initio* no-core Gamow shell model (NCGSM).
  - ✓ The calculated Energy and Width of the Tetraneutron in NCGSM are both close to the recent experimental data (*Nature* 606, 678 (2022)).
  - ✓ The energy and width of Trineutron are both smaller than Tetraneutron in our NCGSM, suggesting that Trineutron would be more possible to be observed than Tetraneutron.
2. The resonant  $T = 1$   $J^\pi = 2^-$  isobaric triplet states of  $A = 4$  nuclei ( ${}^4\text{H}$ ,  ${}^4\text{He}$  and  ${}^4\text{Li}$ ) are investigated with NCGSM.
  - ✓ More precise experimental measurements are urgently needed.

*Thank you for your attention*